

Photons as Geodesics on S^3 : A Topological Correction to the QFT Propagator

Mark Tracy

Boston University mrktracy@bu.edu

March 2026 (*Draft — v3*)

Abstract

The *Imagination Machine* series establishes that the incompatibility of quantum field theory (QFT) and general relativity (GR) is a topological necessity: QFT is a two-Noether-charge theory operating below the minimum toroidal condition, while GR is a three-Noether-charge theory that saturates the topological capacity of S^2 . The resolution condition requires that any successful theory of quantum gravity assume S^3 as the global topology of the containing manifold and treat spacetime topology—not geometry—as the fundamental dynamical variable. We take that condition seriously at the level of the photon propagator. If the containing manifold is S^3 , photons trace geodesics on a curved manifold whose nodes—black holes, in the language of the Cosmic Graph Theorem—are topological gates that constrain and terminate propagation. We derive a corrected position-space photon propagator incorporating geodesic distance on S^3 and a topological sink factor encoding the nodal structure of the cosmic graph. The sink factor is weighted by event horizon area rather than mass, grounding it in the topological capacity of each gate as established by the Bekenstein-Hawking entropy formula and the no-hair chain of TIM XXVIII. The standard Minkowski result is recovered exactly in the limit $R \rightarrow \infty$ with no sinks. The corrected propagator predicts measurable deviations at cosmological separations tracing the filamentary structure of the cosmic web, without new particle content. Black holes constrain the path integration of light itself: the cosmic web is not merely the large-scale structure of matter. It is the large-scale structure of the path integral of light.

Contents

1	The Diagnostic	2
2	The Resolution Condition: Global S^3	2
3	The Corrected Photon Propagator	3
4	Physical Consequences	4

5	Open Questions	5
6	Conclusion	5

1 The Diagnostic

The standard diagnosis of the QFT–GR incompatibility is technical: perturbative quantum gravity is non-renormalizable. TIM XXXI establishes that this diagnosis is wrong at the level of framing. The problem is topological.

The event horizon of a stationary black hole is homeomorphic to S^2 . The topology of S^2 , together with asymptotic flatness, constrains the isometry group of the exterior spacetime to $\mathbb{R} \times U(1)$. By Noether’s theorem, this yields exactly three conserved quantities: mass M , angular momentum J , and charge Q . Three is the *Noether capacity* of S^2 .

QFT discards one of the three axes. Canonical quantization promotes $\phi(\mathbf{x})$ and $\pi(\mathbf{x}) = \partial\mathcal{L}/\partial\dot{\phi}$ to operators,

$$[\phi(\mathbf{x}, t), \pi(\mathbf{y}, t)] = i\hbar \delta^{(3)}(\mathbf{x} - \mathbf{y}), \tag{1}$$

while fixing spacetime as a prior demarcational commitment. QFT is a two-Noether-charge theory. GR, via

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{2}$$

treats all three axes as dynamical. It is a three-charge theory—toroidally complete.

Theorem 1.1 (Tracy Theorem of Topological Incompleteness, TIM XXXI). *The incompatibility of QFT and GR is a topological necessity. No perturbative correction within QFT’s two-axis framework can produce toroidal behavior, because perturbative corrections operate within the existing topology and cannot change it.*

The known pathologies—non-renormalizability, the information paradox, the firewall paradox—are the missing third Noether charge asserting itself at the boundary of QFT’s demarcational commitment.

2 The Resolution Condition: Global S^3

TIM XXXI derives a necessary resolution condition: any successful theory of quantum gravity must (1) treat all three Noether charges as dynamical with no fixed background, (2) not permit Banach–Tarski decompositions, and (3) reproduce QFT and GR as limits. The Closing Loop Theorem (TIM XIX) identifies the required global topology through two independent routes.

Route I (Epistemological). The observational boundary of any embedded epistemic system is S^2 . Its containing manifold is $S^3 = \{x \in \mathbb{R}^4 : \|x\| = R\}$, whose center is the geometric correlate of the view from nowhere—unreachable from within. The Nabaala

Theorem bounds the maximum self-classification depth for a genus- g observer at $H(g) - 1$, where

$$H(g) = \left\lfloor \frac{7 + \sqrt{1 + 48g}}{2} \right\rfloor. \quad (3)$$

For $g = 0$: depth ≤ 3 .

Route II (Physical). Fock (1935) showed that the hydrogen atom in momentum space is a free particle on S^3 , via the stereographic projection

$$u_i = \frac{2p_0 p_i}{p^2 + p_0^2}, \quad u_4 = \frac{p^2 - p_0^2}{p^2 + p_0^2}, \quad p_0 = \sqrt{-2m_e E_n}. \quad (4)$$

The $k = +1$ FRW solution to Einstein's equations independently sources the same S^3 as the spatial section of the containing manifold. Both routes originate in the same geometry. The universe organizes matter and knowledge by the same topology.

The resolution condition is therefore: *assume S^3 globally, and derive everything else from the constraints that topology imposes.*

3 The Corrected Photon Propagator

The standard massless photon propagator in position space (Feynman gauge, flat Minkowski) is

$$D^{\mu\nu}(x - y) = \frac{-i g^{\mu\nu}}{4\pi^2 (x - y)^2}. \quad (5)$$

This assumes a flat background, no global curvature, and propagation to asymptotic infinity. All three assumptions are violated if the containing manifold is S^3 and black holes are topological gates.

Geodesic distance on S^3 . On the three-sphere of radius R , the geodesic distance is

$$d_{S^3}(x, y) = R \arcsin\left(\frac{|x - y|}{2R}\right). \quad (6)$$

This replaces the flat interval in (5). As $R \rightarrow \infty$, $d_{S^3}(x, y) \rightarrow |x - y|$ and the flat result is recovered exactly.

Topological sink factor. The Cosmic Graph Theorem (TIM XXXIII) identifies black holes as nodes $v_i \in V$ of the cosmic graph $G = (V, E, F)$: topological gates whose event horizons, homeomorphic to S^2 , encode the Noether capacity of the boundary. TIM XXVIII establishes that the constraining power of a gate is determined not by its mass alone but by the topological capacity of its boundary, which is measured by the area A_i of its event horizon. By the Bekenstein-Hawking formula,

$$S_i = \frac{A_i}{4\ell_P^2}, \quad A_i = 4\pi r_s^{(i)2} + (\text{spin and charge corrections}), \quad (7)$$

the horizon area is the direct measure of the information—and therefore topological capacity—of the gate. We accordingly weight each node by its effective topological radius

$$\lambda_i = \sqrt{\frac{A_i}{4\pi}} = r_{\text{eff}}^{(i)}, \quad (8)$$

rather than its Schwarzschild radius alone. The topological sink factor is then

$$\mathcal{G}(x, y) = \prod_i \left(1 - \frac{\lambda_i}{d_{S^3}(x, v_i)}\right) \left(1 - \frac{\lambda_i}{d_{S^3}(y, v_i)}\right). \quad (9)$$

This factor vanishes as either endpoint approaches a node (photon captured by the gate), approaches unity in void regions (free propagation), and distinguishes gates by their topological capacity rather than their mass.

The corrected propagator.

$$D^{\mu\nu}(x, y) = \frac{-i g^{\mu\nu}}{4\pi^2 d_{S^3}(x, y)^2} \cdot \mathcal{G}(x, y) \quad (10)$$

4 Physical Consequences

Local scales. Between nodes, $\mathcal{G} \approx 1$ and the geodesic distance approximates the flat interval. The corrected propagator is indistinguishable from (5) at subcosmological separations. QFT works precisely where its demarcational commitment is valid.

Cosmological scales. At separations comparable to inter-node distances, both corrections become significant. The propagator is no longer translation-invariant: it depends on where in the cosmic graph the source and receiver are located.

Lensing without dark matter. TIM XXXIII identifies dark matter as the relational edge structure of the cosmic graph—gravitationally present at every scale but not encodable as local particle content on the observer’s S^2 boundary. The corrected propagator provides a natural mechanism for anomalous lensing: photon geodesics are deformed by the nodal structure of the graph, producing apparent mass concentrations at filament intersections without new particle content. The area-weighted sink factor sharpens this prediction: the lensing signature of a node scales with its horizon area, not its mass, making Kerr and Schwarzschild black holes of equal mass distinguishable by their effect on the propagator.

The path integral of light. The corrected propagator is not a sum over paths in empty space. It is a sum over paths in a graph — where the nodes are black holes, the edges are the filamentary structure of the cosmic web, and the topology of S^3 sets the geodesic distances along which those paths are measured. Black holes do not merely curve the paths of photons. They constrain the path integration of light itself. The cosmic web is not merely the large-scale structure of matter. It is the large-scale structure of the path integral of light.

The $i\epsilon$ prescription reinterpreted. In the flat propagator, $i\epsilon$ handles the on-shell pole at $k^2 = 0$. In the corrected propagator, the sink factor \mathcal{G} replaces this role at topological boundaries: a photon does not pass through a node—it terminates there. The gate preserves only what the topology of its boundary can encode, and nothing more (TIM XXVIII, Theorem 4.4).

Hawking radiation and the maximally conservative completion. TIM XXVIII establishes that the Hawking radiation spectrum is the maximally conservative completion of partial structure: the thermal distribution is the most uncertain distribution consistent with the known invariants (M, Q, J) , breaking no additional symmetry beyond what the constraints require. The area-weighted sink factor is consistent with this: it assigns to each node exactly the topological weight its boundary can carry, neither more nor less.

5 Open Questions

1. **Momentum-space form.** The sink factor and geodesic distance break translational invariance. Computing the Fourier transform and identifying the modified pole structure is the most immediate technical task.
2. **The S^3 identification.** TIM XIX notes that Fock’s S^3 lives in momentum space while the FRW S^3 is spatial. A rigorous identification of these two objects would substantially strengthen the framework.
3. **Quantitative predictions.** The lensing deviations are in principle computable from the known large-scale structure of the cosmic web. The area-weighted sink factor generates a specific prediction: lensing signatures should correlate with horizon area rather than mass, distinguishing spinning from non-spinning black holes at cosmological scales.
4. **Multi-photon amplitudes.** How the sink factor enters multi-photon diagrams, and whether the resulting amplitudes remain unitary, connects to the information paradox resolution of TIM XXXI.
5. **Dynamical R .** The corrected propagator treats R as a fixed FRW parameter. A fully toroidally complete propagator would treat the topology of S^3 itself as dynamical—the remaining gap between this result and the full resolution condition.
6. **Spin and charge corrections to λ_i .** The effective topological radius is defined here via the horizon area of the Kerr-Newman solution. A full treatment would express λ_i explicitly in terms of (M_i, Q_i, J_i) and examine how spin and charge independently modulate the sink factor.

6 Conclusion

The incompatibility of QFT and GR is topological. The resolution condition requires global S^3 and the treatment of spacetime topology as the fundamental dynamical variable. Taking that

condition seriously at the level of the photon propagator yields a position-space correction that recovers flat Minkowski as a local limit, predicts deviations at cosmological scales tracing the nodal structure of the cosmic graph, and provides a topological mechanism for dark matter lensing signatures without new particle content. The refinement of the sink factor—from Schwarzschild radius to event horizon area—grounds the propagator correction in the topological capacity of each gate as established by TIM XXVIII, and generates a new distinguishing prediction: lensing scales with horizon area, not mass.

None of this required new physics. It required taking the topology that was already there seriously all the way down to the propagator — and recognizing that black holes do not merely curve the paths of photons. They constrain the path integration of light itself. The cosmic web is not merely the large-scale structure of matter. It is the large-scale structure of the path integral of light.

References

- [1] M. Tracy et al. *The Imagination Machine: A Compendium* (TIM I–XXXIII). Boston University, 2026.
- [2] V. Fock. Zur Theorie des Wasserstoffatoms. *Zeitschrift für Physik*, 98(3–4):145–154, 1935.
- [3] E. Noether. Invariante Variationsprobleme. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, pages 235–257, 1918.
- [4] J. D. Bekenstein. Black holes and entropy. *Physical Review D*, 7(8):2333–2346, 1973.
- [5] S. W. Hawking. Particle creation by black holes. *Communications in Mathematical Physics*, 43(3):199–220, 1975.
- [6] R. Penrose. Gravitational collapse and space-time singularities. *Physical Review Letters*, 14(3):57–59, 1965.