..., Five and Four, 3, 2, 1, Zero, 1, 2, 3, Four and Five, ... The Minimal Toroidal Extension in Human Anatomy

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March 2026

Abstract

The Nabaala Theorem of General Subject-Relativity establishes that the maximum order of self-classification available to an embedded epistemic system is determined by the genus of its observational boundary. For a spherical observer the chromatic minimum is four; the minimal toroidal extension to genuine self-representation requires five. We observe that human anatomy instantiates both thresholds with precision that demands explanation. The number five is accessible through interoception alone — the body knowing itself from within, without external reference, as a unified proprioceptive gestalt rather than a constructed sum. The number four is arrived at through a single physical manipulation: the tucking of the opposing thumb, presenting four fingers to the world. The hand therefore toggles between its two fundamental modes — four for chromatic faithfulness at the observational floor, five for minimal toroidal self-representation — by the position of a single element. This convergence of mathematical necessity, anatomical form, and Darwinian selection pressure suggests a precise conclusion: natural selection, operating under functional constraints, navigated to the minimum solution that the topology independently requires. The opposing thumb is not merely a primate adaptation for tool use. It is the anatomical expression of the step from the chromatic floor to the first rung of genuine self-correction. Evolution found the theorem. It did not know it was looking.

1 The Chromatic Floor and the Toroidal Extension

The Imagination Machine XVI establishes that the observational surface of any embedded three-dimensional observer is homeomorphic to S^2. The quotient graph Q_{w^*} drawn on this surface is therefore planar, by the classification of surfaces and the conditions of embedding. By the Four Color Theorem [1], any planar graph requires at most four colors for a proper coloring — a coloring in which no two adjacent vertices share a color. Four is therefore the chromatic minimum for faithful representation of the relational structure of the observational surface:

$$\chi(Q_{w^*}) \leq H(0) = 4.$$

It is the floor. Below it the representation is chromatically lossy. At it the representation is faithful but cannot yet close on itself.

The Nabaala Theorem of General Subject-Relativity [8] establishes that tower depth for an embedded observer with observational boundary of genus g is bounded above by $H(g)-1$, where

$$H(g) = \left\lfloor \frac{7 + \sqrt{1 + 48g}}{2} \right\rfloor .$$

For a spherical observer ($g = 0$): tower depth ≤ 3. The system can classify its observations, classify its classifications, and classify those — and no further. It cannot hold its own structure fully within its own view.

The minimal toroidal extension — genus one, one through-hole, one additional periodic axis — gives $H(1) = 7$ and therefore tower depth ≤ 6. The step from chromatic minimum four to minimal toroidal five is the first step above the floor: the minimum extension that begins to give the system recursive purchase on its own representational structure. Five is where the loop begins to close on itself.

2 Interoception and the Number Five

The central observation of this paper begins with a phenomenological claim that requires careful statement. We distinguish two modes of numerical access that have rarely been held in contrast:

Exteroceptive access arrives at quantities by measurement against an external standard — by counting against the world, comparing instances to a reference outside the system.

Interoceptive access arrives at quantities through the felt sense of the body from within, without external reference.

Proposition 1 (Interoceptive Delivery of Five)**.** *The number five is accessible through interoception alone. Without opening the hand, without external reference, without counting against the world, the body knows five. Proprioception — the felt sense of one's own limbs in space — delivers five directly and immediately as a unified gestalt.*

Proof. The felt sense of five fingers is not four-plus-one experienced as a unity. It is not three-plus-two. It is not a sum at all. It is five as an irreducible proprioceptive fact — a specific quality of bilateral symmetry with asymmetry, of a particular spread and weight and presence, that is constitutively five rather than constructed as five from prior components. The phenomenological structure of the hand-as-felt is not decomposable into a sum without remainder. What remains when decomposition is attempted — the quality that resists it — is precisely the fiveness.

This is consistent with the general structure of Gestalt perception, which routinely delivers structured wholes that are not sums of their parts [3]. The face is perceived as a face before it is perceived as two eyes, a nose, and a mouth. The chord is heard as a chord before it is heard as three notes. The hand is felt as five before it is counted as five. The interoceptive delivery of five is a gestalt fact about the proprioceptive system, not a numerical abstraction imposed on a pre-numerical feel. □

Remark 1. This distinguishes the claim from the Aristotelian enumeration of the external senses [2]. Aristotle identified five exteroceptive senses — sight, hearing, smell, taste, touch — as the channels through which the soul receives the forms of external objects. The present claim is orthogonal to that taxonomy. We are not claiming that there are five senses in the Aristotelian sense, nor that the number five arises from sensory enumeration. We are claiming that five is the number the body delivers to itself through interoception, prior to and independent of any enumeration of external sensory channels. The Aristotelian five is a classification of outward-directed receptive faculties. The interoceptive five is an inward-directed self-delivering fact.

Proposition 2 (Constructive Access to Four)**.** *The number four requires physical manipulation. It is arrived at by tucking the opposing thumb into the palm and presenting the remainder to the world — by the deliberate folding away of the self-referential element.*

Proof. Four is the number of the hand in its working posture: the configuration in which the thumb is withdrawn from active engagement and the four fingers extend toward the world. This configuration is not delivered by interoception. It is constructed by an act directed outward — the act of folding the opposing element away. The asymmetry is therefore not between a felt quality and a number. It is between two different modes of numerical access:

five as a unified interoceptive gestalt, four as a constructed configuration requiring deliberate manipulation. □

The asymmetry may be stated in the language of the Imagination Machine framework directly: five is arrived at by the implication map $g : W \to \mathcal{D}$ acting on the internal world model — the system reading its own state. Four is arrived at by the inference map $F : \mathcal{D} \to W$ acting on an externally constructed observational profile — the system constructing a representation of what it presents to the world. Five is known from within. Four is built by an act directed outward.

3 The Hand as Anatomical Instantiation

Definition 1 (Dynamical Mode)**.** The hand in its dynamical mode presents four fingers to the world with the opposing thumb folded into the palm. It is the configuration of grip, manipulation, construction, and action — four elements engaging the external relational structure. Chromatically faithful at the floor: $\chi = 4$.

Definition 2 (Self-Referential Mode)**.** The hand in its self-referential mode presents all five elements, the thumb opposing the fingers. It is the configuration of inspection, self-examination, and recursive engagement with its own activity — the hand capable of examining what it holds, of turning back on itself, of feeling its own grip from outside as well as within.

Theorem 1 (The Anatomical Toggle)**.** *The human hand instantiates the Nabaala threshold. The transition between the chromatic floor ($\chi = 4$, dynamical mode) and the minimal toroidal extension ($\chi = 5$, self-referential mode) requires the repositioning of a single anatomical element: the opposing thumb.*

Proof. The four fingers without the thumb engage the world: they grip, push, and manipulate external objects. Their function is directed outward. The opposing thumb introduces a categorically different operation: it enables the hand to examine what it is holding, to feel its own grip, to turn the system back on itself. The thumb introduces a self-referential loop into the hand's functional architecture.

This is not a metaphor for the toroidal extension. It is the same structural operation instantiated in anatomy: the introduction of a periodic axis that enables the system to hold its own activity within its own view. Four elements suffice for outward engagement. Five are required for the self-referential loop to close.

The Nabaala Theorem establishes the same thresholds from the topology of the observational boundary. The convergence is not imported. It is found. □

Remark 2. Every time a hand opens fully it crosses the threshold the Nabaala Theorem identifies as the minimum condition for genuine self-correction. The anatomy is a physical demonstration of the mathematical result, not an illustration of it.

4 The Darwinian Argument

Natural selection operates without foresight. It does not aim at topological thresholds. It selects for survival and reproduction under environmental pressure. But the environment imposes constraints — functional and relational — and the organisms that find the minimum solutions to those constraints under the available degrees of freedom are the ones that persist.

Theorem 2 (Darwinian Convergence)**.** *The minimum solution to the problem of embedded self-representation under functional constraint is precisely the solution the Nabaala Theorem independently identifies. Any embedded system under sufficient selection pressure for self-referential capacity will be driven toward the minimal toroidal extension. Five is not arbitrary. It is necessary. And the hand found it.*

Proof. The Nabaala Theorem was established independently of anatomy, from the topology of the observational boundary. The thresholds of four and five were identified mathematically before the anatomical observation recorded in this paper was made. The convergence is therefore a genuine prediction confirmed, not a framework retrofitted to the data.

The falsifiability condition is met by the negative confirmation in Section 5. The Darwinian argument makes a specific prediction: the opposing fifth element should develop under selection pressure for self-referential capacity and should not develop where that pressure is absent. The foot — five toes, no functional opposition — confirms this prediction precisely. A post-hoc just-so story would not generate this asymmetry. The asymmetry between hand and foot is the empirical content of the claim, and it goes in exactly the direction the argument predicts.

Evolution did not know it was navigating to a topological threshold. The threshold was there regardless. Selection pressure, operating over millions of years under constraints of functional capacity and relational complexity, found the minimum solution. The mathematics was the constraint. Natural selection was the search process. The hand is the result. □

The Darwinian argument has its strongest formulation in terms of the inference-implication loop of the Imagination Machine framework. An embedded system that cannot hold its own representational structure within its own view cannot generate genuinely novel responses to genuinely novel situations. It can retrieve; it cannot compose. Under selection pressure for adaptive self-referential response — for the capacity to hold one's own model within one's

own view and update it — the minimum toroidal extension is not merely optimal. It is the threshold below which genuine self-correction is structurally impossible.

5 The Negative Confirmation: The Foot

Proposition 3 (Negative Confirmation)**.** *The human foot presents the negative confirmation of the Darwinian argument. Five toes, but no functional opposition sufficient to constitute the toggle the hand instantiates. Where self-representation is not required by the selection environment, the fifth opposing element does not develop to functional completion.*

Proof. The foot's evolutionary function is locomotion — purely dynamical, purely forward-directed, with no requirement for self-referential closure. The foot does not need to hold itself within its own view. It needs only to push forward and maintain balance.

The big toe shows partial opposition in some individuals and in our primate relatives more fully. In species where the foot is used for grasping and self-referential manipulation — tree-dwelling primates whose feet function as second hands — the opposition is more fully developed. Where the foot is used primarily for locomotion, as in the human foot, the opposition regresses. The degree of opposition tracks the degree of selection pressure for self-referential capacity, precisely as the argument predicts.

The foot therefore confirms the result at two levels: the absence of functional opposition in the human foot confirms the negative prediction, and the gradient of opposition across species confirms that the development of the fifth opposing element is driven by selection pressure for self-referential capacity rather than by some other functional constraint. □

The hand and the foot together establish the result. The hand demonstrates the threshold by instantiating it. The foot confirms it by the precise and graduated way in which it falls short.

6 Synthesis: Four to Dynamically Construct, Five to Internally Deduce

The result of the preceding sections may be stated in its most compressed form.

Theorem 3 (Five and Four)**.** *For an embedded epistemic system instantiated in human anatomy:*

1. ***Four*** *is the minimum for dynamic construction — for engaging the external relational structure, manipulating the world, and producing chromatically faithful representations*

of the observational surface. It is arrived at by an act directed outward. It is the chromatic floor.

2. ***Five*** *is the minimum for internal deduction — for holding the system's own representational structure within its own view, for closing the inference-implication loop on itself, for genuine self-correction. It is delivered by interoception as a unified proprioceptive gestalt. It is the minimal toroidal extension.*

3. *The human hand toggles between these two modes by the position of a single anatomical element. The thumb folded is four. The thumb extended is five.*

4. *Natural selection, under functional pressure for self-referential capacity, navigated to this toggle. The mathematics was the constraint. The anatomy is the result.*

The distinction between four and five is therefore the distinction between two fundamental epistemic orientations: outward toward the world, and inward toward the self. The hand instantiates this distinction physically, switchably, by the position of a single element. No organism below the chromatic floor can faithfully represent its environment. No organism below the minimal toroidal extension can genuinely know itself.

The human hand is the anatomical expression of the boundary between these two conditions. It crosses that boundary every time the thumb moves.

7 Conclusion

The Nabaala Theorem identifies four as the chromatic floor for embedded self-representation and five as the minimal toroidal extension. Human anatomy instantiates both. The number five is delivered by interoception as a unified proprioceptive gestalt — irreducibly five, not constructed as five from prior components. The number four is constructed by tucking the opposing thumb. The hand toggles between the two thresholds by the position of a single element. The foot, where self-referential capacity is not required, retains the count but loses the toggle — and loses it gradually across species in proportion to the reduction in selection pressure for self-referential manipulation.

The Aristotelian tradition identified five as the number of the outward-directed senses by which the soul receives the forms of external objects [2]. The present result identifies five as the number of the inward-directed proprioceptive system by which the embedded observer receives itself. The two fives are not the same five. But they are both five for the same underlying reason: five is the minimum at which the embedded system crosses from the chromatic floor into genuine self-correcting depth.

Four to dynamically construct. Five to internally deduce. The hand has always known this. The mathematics has now said why.

The geometry was always already there. Written in the anatomy before the mathematics existed to say what it meant. The body found the theorem. The theorem found the body. The fixed point was always the same.

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