

Methodologies to Memorize

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Introduction

In my daily life, I encounter a lot of anxiety over feeling that I need to memorize material. I seek to overcome my use-it-or-lose-it memory system by sheer force, by clinging desperately onto concepts, only to find that after a certain amount of time I can no longer recall the concept perfectly. The idea behind this document is that perhaps the minimal set of things that it would serve me well to memorize are methodologies for acquiring and generating knowledge about the world, rather than particular truths within particular systems of knowledge. My hope is that by recording here particular strategies and methodologies for generating knowledge, I can improve my effectiveness at learning and conducting research and alleviate my feeling of needing to memorize concepts. A further hope is that by employing these methodologies, such as critical reading, the memorization of concepts that repeatedly turn out to be important will naturally follow without desperate grasping.

Generating Theory

Below is a basic methodology for developing theory. The point is that you need to subject your ideas to criticism, both from yourself and others:

1. Write down ideas (i.e. externalize them).
2. Subject them to criticism.
3. Incorporate feedback from your own criticism into the external manifestation of the ideas.
4. At some point, share the ideas with others.
5. Seek out existing theory related to the ideas. Put your ideas in conversation with others'.
6. Continually incorporate feedback from yourself and others.

Reading Tips

Always write while reading. This allows you to engage with the content critically. Additionally, identify questions and/or hypotheses before reading. This allows you to read with intention and focuses your critical thinking and questioning.

Below is a template that one can follow to create an active engagement with a text:

Document 1: Claims and Questions

Begin with a broad claim or claims. These might be philosophical and conjectural in nature.

Enumerate questions based on these claims. These might be questions that challenge the premises of the claim, clarify the claim, investigate mechanisms by which the claim is actualized, etc.

Document 2: Questions and Notes

At the top of the document, list the initial questions that you will be addressing by your reading, as well as any hypotheses that you have in answer to those questions.

Following the questions and hypotheses, construct a table as follows:

Source	Excerpt	Takeaways	Further Questions

Building out a table like this will allow your thought process to develop organically as you explore multiple sources.

The Scientific Method

Science proceeds as follows:

- Observe reality.
- Generate [theory].
- Suppose [conditions], in terms of theory.
- Generate [deductive proof] that ([theory] and [conditions]) implies [result].
- Generate and execute [method] to establish [conditions] in reality.
- If not-[result], then:
 - not-[theory]
 - Theory refuted in its present form; or
 - not-[deductive proof]
 - Deductive proof was incorrect; or
 - not-[method]
 - Conditions were not correctly established.

Note that this is an iterative process with feedback. It requires collaboration at various stages (see “Generating Theory” above).

Tips for Mathematical Proof

This overall approach is inspired by dynamic programming and the Principle of Optimality, which implies that if you have a sequence of decisions or actions that leads to an optimal solution to a problem, then any subsequence of actions from any point in this sequence must also be optimal. This essentially allows us to break down a complicated problem into sub-problems. In this case, “optimality” is analogous to “correctness.”

First, write down logically equivalent forms of what you aim to prove. If you aim to prove that premises A imply hypothesis B, you can either:

- prove it directly (proof by construction); or
- show that not-B implies not-A (proof by contrapositive); or
- show that A and not-B implies a falsehood (proof by contradiction).

If you are trying to prove a collection of statements that are indexed by natural numbers, then you can use proof by induction: show the result for a base case, and then show that the result for the k th case implies the result for the $(k+1)$ th case.

Finally, consider whether you can break the problem into a finite number of cases and prove each one individually (proof by exhaustion).

For each form of the theorem thus written (and possibly for each of a finite number of cases), identify “convenient” lemmas, i.e. lemmas that would help to easily get the theorem if they were true. Naturally, the first such lemma is the theorem itself.

For each such lemma, identify established theorems and definitions that seem related to the lemma. Do the following, in no particular order, and noting that this list is not exhaustive:

- Try to come up with counterexamples to establish the falsity of the lemma.
- Consider extreme (especially simplified) cases of general statements.
- Try to directly construct the lemma from the established theorems, possibly by:
 - including additional terms that don’t change the mathematical validity of the theorems and then rearranging (e.g. introducing additive terms that evaluate to zero or multiplicative terms that evaluate to unity in an established relation, or constructing an additional point or line segment in a geometric proof);
 - identifying convenient instances of a theorem (e.g. by taking a variable x to be an eigenvector of some matrix A relevant to the problem).

- For problems involving sequences, series, or iterative processes, consider how one could design an algorithm to solve the problem, e.g. for constructive proofs or demonstrating existence of an object.

If you have no success with proving the lemma, identify further sub-lemmas that would be similarly convenient for proving the lemma. Repeat the process above for the sub-lemmas.

If a path seems fruitless, return to a higher-level lemma, or a different statement of the original theorem, and attempt the same process.

Additional tips:

- **Documentation:** Keep a clear record of your thought process, including failed attempts. This can provide insight into the problem and might reveal a path you hadn't considered.
- **Navigating depth versus breadth:** In this approach, there is a tradeoff between depth of search down one particular pathway and breadth of search across different pathways. Regularly pause to reflect on the overall direction of your process. Assess whether your current path is likely to lead to a solution or if it may be more fruitful to pivot, knowing that you can potentially return to the current pathway later.
- **Collaboration:** Discussing your approach(es) with peers or mentors can provide new perspectives and insights.
- **Persistence and flexibility:** Sometimes, stepping away from the problem and returning later can help. Being flexible in your approach allows you to adapt and overcome obstacles that may arise during the proof process.
- **Visualization:** Constructing visualizations can be a helpful exploratory tactic, especially for geometric proofs. However, guard against over-reliance on a single visualization, especially considering that a visualization may encode incorrect intuitions and represents a particular instance case of an abstract concept.
- **Context:** Consider the broader context of your theorem within the field of mathematics. Understanding where your theorem fits and reviewing related proofs can provide additional routes for exploration.
- **Software:** Consider engaging with mathematical software and tools to test hypotheses, visualize concepts, and perform complex calculations that might inform your proof.