The Imagination Machine XV: Chromatic Number and the Sensory Constraint on Embedded Observers

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Abstract

The *Imagination Machine* series establishes that embedded epistemic systems cannot attain a view from nowhere: every observational surface is local. The present paper derives a quantitative consequence of this constraint. We take as our central assumption the geometric picture introduced in *The Imagination Machine XIV*: an embedded observer inhabits a three-dimensional manifold understood as a cross-section of a four-dimensional hypersphere, so that the local observational surface is homeomorphic to the two-sphere S^2. Every finite graph drawn on S^2 is planar. Two classical results then apply. The Five Color Theorem — provable from Euler's formula alone — establishes that the quotient graph induced on the observational surface by any admissible world model is five-colorable. The Four Color Theorem tightens this to four.

We interpret these bounds within the longstanding question of how many senses an embedded observer possesses. The classical enumeration, stable from Aristotle through the early modern period, identifies five. Modern sensory biology has pressed the count upward, identifying proprioception, vestibular sensation, thermoception, nociception, interoception, and further modes depending on the criteria of individuation. We argue that this three-way structure — a stable classical five, a tighter non-constructive four, and an open-ended upward pressure — is explained without remainder by the chromatic structure of the framework. Five is the constructive chromatic bound on the observational surface, explaining the stability of the Aristotelian count. Four is the tight bound, non-constructively established, explaining the minority tradition that has sought to reduce the classical enumeration. The upward pressure of modern sensory biology corresponds to ascending the simplicial tower above the observational

surface, where the chromatic structure is no longer bounded by the planarity of S^2 and new distinguishing modes become individuable at each order. Neither the classical count nor its modern proliferation is empirically arbitrary; both are structural consequences of the embedding geometry.

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1 Introduction

Imagine a bubble around your body that follows you wherever you go. You cannot step outside it. Every piece of data that reaches you passes through its surface, and as it does, your mind structures it as relational information — connections between nodes, a graph. The bubble is two-dimensional; the world that generates the data is four-dimensional. A four-dimensional graph projected onto a two-dimensional surface becomes a planar graph. And the minimum number of colors needed to properly color any planar map — so that no two adjacent regions share a color — is four. Historically the constructive argument gave five. This, we will argue, is why the minimum number of irreducible discriminating modes available to an embedded observer is four or five: not because of anything contingent about human anatomy, but because of the geometry of being inside.

The *Imagination Machine* series begins from the constraint that an epistemic system embedded within the world has no access to an external vantage point. Knowledge must therefore be defined not as correspondence with an independently accessible outside, but as the stabilization of representations through the internal closure of the inference–implication loop. This founding constraint has been developed formally through fixed-point conditions on world models, the inclusion of classifiers within the observation space, the quotient structure of representational compression, and, in the later papers of the series, a geometric picture in which the embedded observer inhabits the surface of a hypersphere.

The present paper derives a quantitative consequence of that picture. If the observational surface is the two-sphere S^2 — as the geometry of *The Imagination Machine XIV* implies — then the topology of that surface imposes a constraint on the minimum representational resources any embedded observer requires at that surface. The constraint is not a contingent feature of human anatomy or evolutionary history. It is a consequence of planarity.

The argument proceeds in two steps of unequal difficulty. The first is internal to the series: planarity of the observational quotient graph is derived from the hypersphere geometry and the graph-theoretic realization of compression established in *The Imagination Machine XI*. The second invokes two classical results. The Five Color Theorem, whose proof we sketch from Euler's formula, establishes that five distinguishing resources constructively suffice. The Four Color Theorem, which we cite rather than reprove, tightens this to four.

These results make contact with a genuine and longstanding question: how many senses does an embedded observer have? The question has three historically distinct answers. Aristotle identified five — sight, hearing, smell, taste, and touch — and this enumeration remained the dominant account for over two millennia. A minority tradition, sharpened by functionalist analysis, has sought to reduce the count, noting that some of the classical five can be partially analyzed in terms of others. Since Sherrington's identification of proprioception at the turn of the twentieth century, modern sensory biology has pressed in the opposite direction, now recognizing upward of twenty distinct sensory modes depending on the criteria of individuation.

We argue that this three-way structure is exactly what the chromatic framework predicts. Five is the constructive upper bound on the observational surface: it follows from Euler's formula and exhibits an explicit algorithm. This explains the stability of the Aristotelian count — five is the number a reflective enumerator arrives at by working through the observational surface constructively. Four is the tight non-constructive bound: it requires the Four Color Theorem and is not arrived at by enumeration alone. The upward pressure of modern sensory biology corresponds to ascending the simplicial tower above the surface, where planarity no longer constrains the chromatic structure and new distinguishing modes become individuable at each order.

Section 2 recalls the relevant machinery from the series. Section 3 states the geometric assumption and derives planarity. Sections 4 and 5 establish the two chromatic bounds. Section 6 addresses the upward pressure via the simplicial tower. Section 7 assembles the three-level account of the senses. Section 8 discusses implications and open questions.

2 Background from the Series

We recall the formal elements required for the present argument. Full treatments appear in the cited papers.

2.1 The Observational Surface and Quotient Structure

The Imagination Machine I introduces the observation space D equipped with a probability structure (D, Σ_D, μ_D). Each world model w induces a classifier $\omega_w \colon D \to Z_w$ that partitions D into equivalence classes: $d_1 \sim_w d_2$ if and only if $\omega_w(d_1) = \omega_w(d_2)$. The quotient space $Q_w = D/{\sim_w}$ is the compressed representation of observations under the model. Self-consistent world models are fixed points of the operator $T = F \circ g$, where $F \colon D \to W$ is the inference map and $g \colon W \to D$ is the implication map. We write w^* for an arbitrary such fixed point.

2.2 The Graph-Theoretic Realization

The Imagination Machine XI establishes that graph theory provides the natural concrete realization of the compression–extension architecture. The observation space is realized as a graph $G = (V, E)$ with vertices representing entities and edges representing binary relations. Compression is graph quotient: given an equivalence relation $\sim$ on V, the quotient graph $G/{\sim}$ has vertex set $V/{\sim}$ and edge set

$$\{([u],[v]) : \exists\, u' \sim u,\ v' \sim v \text{ with } (u',v') \in E \text{ and } [u] \neq [v]\}.$$

The quotient map $q\colon G \to G/{\sim}$ is a graph morphism. The equivalence relation $\sim_w$ is the graph-theoretic instance of the model-induced equivalence on D.

2.3 The Simplicial Tower

The Imagination Machine X identifies the common simplicial backbone underlying the compression–extension operations of the series. The clique complex $X(G)$ of a graph G has as its k-simplices the $(k+1)$-cliques of G. Face maps correspond to compression (dropping a vertex from a clique) and extension operations to higher-dimensional completion. The k-skeleton $X(G)^{(k)}$ consists of all simplices of dimension at most k. Compression at each simplicial order reduces the complex to a lower-dimensional skeleton, and ascending the tower reveals representational structure that is invisible at the surface level.

2.4 The Hypersphere Geometry

The Imagination Machine VIII and *The Imagination Machine XIV* introduce the geometric picture of embeddedness. The three-sphere

$$S^3 = \{x \in \mathbb{R}^4 : \|x\| = r\}$$

is proposed as the maximally conservative geometry for an embedded observer: closed, without boundary from within, and with no accessible center. *The Imagination Machine XIV* identifies the center $0 \in \mathbb{R}^4$ as the geometric correlate of the philosophical ideal of a view from nowhere — the unique point of maximal symmetry with respect to all embedded positions, unavailable to any embedded observer. The observational surface of an embedded observer is the two-dimensional boundary of the locally accessible region from a position on S^3.

3 The Observational Surface is Planar

Assumption 3.1 (Observational Surface)**.** The local observational surface of an embedded epistemic system is homeomorphic to the two-sphere

$$S^2 = \{x \in \mathbb{R}^3 : \|x\| = 1\}.$$

This surface arises as the boundary of the locally accessible observational region for an observer embedded in the three-dimensional manifold S^3.

This assumption follows from the hypersphere geometry of *The Imagination Machine XIV*. An observer at $x \in S^3$ has access to local three-dimensional neighborhoods of x within S^3. As the radius of any such neighborhood tends to the observational horizon, its boundary is homeomorphic to S^2.

Proposition 3.2 (Planarity of the Observational Quotient Graph)**.** *Under Assumption 3.1, the quotient graph Q_{w^*} induced on the observational surface by any admissible world model w^* is planar.*

Proof. The relational structure of the environment is a connected graph embedded in S^3. Such a graph is generically non-planar: S^3 contains K_5 and $K_{3,3}$ freely, with no Kuratowski obstruction.

The observer does not encounter this graph from outside. It encounters the graph at the moment edges cross the observational boundary S^2. At any such moment, what is recorded is the cross-section of the graph with S^2: the configuration of points at which edges pierce the boundary surface, together with the nodes that form wherever edges meet.

This cross-section is inherently planar. Edge crossings in S^3 — one strand passing over another — are depth features: they require a coordinate distinguishing near from far, interior from exterior. The observational surface S^2 is precisely the locus where interior becomes exterior. It carries no depth coordinate; it records only which side each point is on, not the ordering of strands within the interior. Therefore no crossing-over survives passage through S^2. Where two edges that would have crossed in S^3 arrive at the same point on S^2, they meet at a node. From any node, edges extend across the surface without the depth-ordering information that distinguished them in the interior. The cross-section is therefore a graph drawn on S^2 without crossings.

The quotient graph Q_{w^*} is the compression of this cross-sectional observation under the world model w^*. Since the quotient map $q\colon G \to G/{\sim}$ is a graph morphism and graph morphisms do not introduce crossings, Q_{w^*} inherits planarity from the cross-section.

Confirmation. That every finite graph on S^2 is planar also follows from the homeomorphism $S^2 \setminus \{p\} \cong \mathbb{R}^2$ for any point p, and the classical equivalence between planarity and embeddability in S^2 without crossings. The crossing argument above gives the physical mechanism; stereographic projection confirms the mathematical fact. □

Remark 3.3. Planarity of Q_{w^*} is not a contingent feature of any particular world model. It follows from the structure of observation itself: observation is cross-sectional, the cross-section discards depth, and the discarding of depth is what forces planarity. Crossing-overs do not disappear at the boundary — they resolve into nodes. The graph on S^2 is the graph of the environment as seen from the boundary: the same relational content, transformed by the act of crossing. This holds for every fixed point w^* of the inference–implication operator T.

4 The Five Color Bound

We derive the five-color bound constructively from Euler's formula.

Lemma 4.1 (Euler's Formula)**.** *For any connected planar graph $G = (V, E)$ drawn in the plane with F faces (including the unbounded face),*

$$|V| - |E| + F = 2.$$

Lemma 4.2 (Average Degree Bound)**.** *Every planar graph contains a vertex of degree at most* 5*.*

Proof. Assume G is connected with $|V| \geq 3$. In any planar embedding, each face is bounded by at least three edges, and each edge borders at most two faces, so $3F \leq 2|E|$, giving $F \leq \frac{2}{3}|E|$. Substituting into Euler's formula:

$$2 = |V| - |E| + F \leq |V| - |E| + \tfrac{2}{3}|E| = |V| - \tfrac{1}{3}|E|,$$

hence $|E| \leq 3|V| - 6$. The sum of degrees equals $2|E|$, so

$$\frac{2|E|}{|V|} \leq \frac{2(3|V| - 6)}{|V|} = 6 - \frac{12}{|V|} < 6.$$

Since the average degree is strictly less than 6, some vertex has degree at most 5. □

Theorem 4.3 (Five Color Theorem for Embedded Observers)**.** *Under Assumption 3.1, the quotient graph Q_{w^*} admits a proper vertex coloring using at most five colors. Consequently, any embedded epistemic system requires at most five irreducible distinguishing resources to properly differentiate all adjacent observational regions.*

Proof. By Proposition 3.2, Q_{w^*} is planar. We show by induction on $|V(Q_{w^*})|$ that every planar graph is five-colorable.

Base case. Graphs on at most five vertices are trivially five-colorable.

Inductive step. Let G be planar on $n > 5$ vertices, and assume all planar graphs on fewer than n vertices are five-colorable. By Lemma 4.2, G contains a vertex v of degree at most 5. Let $G' = G \setminus \{v\}$; since subgraphs of planar graphs are planar, G' is planar, and by the inductive hypothesis G' admits a five-coloring. Fix such a coloring.

If the neighbors of v use at most four of the five colors, assign v the remaining color.

If the neighbors of v use all five colors, then v has exactly five neighbors $v_1, \ldots, v_5$ (in cyclic order around v in the planar embedding), each receiving a distinct color $1, \ldots, 5$. Consider the subgraph H_{13} induced on vertices colored 1 or 3. If v_1 and v_3 lie in different connected components of H_{13}, swap colors 1 and 3 in the component of v_1; this valid recoloring of G' frees color 1 for v.

If v_1 and v_3 are connected in H_{13}, a path P_{13} in H_{13} from v_1 to v_3, together with the edges vv_1 and vv_3, forms a Jordan curve separating v_2 from v_4 and v_5. Hence v_2

and v_4 lie in different components of the subgraph H_{24} induced on vertices colored 2 or 4. Swapping colors 2 and 4 in the component of v_2 frees color 2 for v.

In all cases the coloring extends to v. □

Remark 4.4. The proof is constructive: it exhibits an explicit five-coloring algorithm. The Kempe-chain step never inspects more than two colors simultaneously and terminates after a bounded sequence of local swaps. This constructive character is significant for the interpretation in Section 7.

5 The Four Color Bound

Theorem 5.1 (Four Color Theorem, Appel–Haken 1976 [1]). *Every planar graph admits a proper vertex coloring using at most four colors.*

This result is cited rather than proved. The original proof proceeds by computer-assisted verification of a finite unavoidable set of reducible configurations and does not admit a short reconstruction. Its proof strategy — reducibility and discharging — is qualitatively different from the Kempe-chain argument of Theorem 4.3, and the gap between them is not merely a gap in proof length but a gap in constructive content.

Theorem 5.2 (Chromatic Constraint on Embedded Observers). *Under Assumption 3.1, the quotient graph Q_{w^*} induced by any admissible world model w^* admits a proper four-coloring. Any embedded epistemic system therefore requires at most four irreducible distinguishing resources to properly differentiate all adjacent observational regions at the observational surface. This bound is tight: there exist planar graphs — hence there exist possible observational configurations on S^2 — requiring exactly four colors.*

Proof. By Proposition 3.2, Q_{w^*} is planar. By Theorem 5.1, every planar graph is four-colorable. Tightness: the complete graph K_4 is planar and requires exactly four colors. □

Remark 5.3. Theorems 4.3 and 5.2 apply to the same object Q_{w^*}. They establish bounds of five and four respectively. The gap between them is not a gap in our knowledge of Q_{w^*}; it is the gap between a constructive bound and a tight bound, established by proofs of qualitatively different character. Both bounds apply to every admissible world model on the observational surface.

6 The Simplicial Tower and Upward Pressure

Theorems 4.3 and 5.2 characterize the chromatic structure at the observational surface: the two-sphere S^2 forces planarity and planarity forces the four- and five-color bounds. But an embedded epistemic system is not confined to representations at the surface level. The simplicial tower of *The Imagination Machine* X extends the representational architecture through ascending orders of the clique complex $X(Q_{w^*})$.

At the k-skeleton $X(Q_{w^*})^{(k)}$, new relational structure becomes visible that is invisible at lower orders. The chromatic structure at order $k > 0$ is determined not by the planarity of S^2 but by the combinatorial structure of the k-skeleton itself. In general, the chromatic number of higher-order skeleta is not bounded by four or five; it grows with the complexity of the clique structure and is not constrained by the surface geometry alone.

Proposition 6.1 (Unbounded Chromatic Growth in the Tower)**.** *For $k \geq 1$, the chromatic number of the k-skeleton $X(Q_{w^*})^{(k)}$ is not in general bounded by the chromatic number of Q_{w^*}. In particular, for any $n \geq 1$ there exist quotient graphs Q_{w^*} and skeleton orders k such that $\chi(X(Q_{w^*})^{(k)}) \geq n$.*

Proof. The clique complex $X(K_n)$ of the complete graph K_n has as its $(n-1)$-simplex the single n-clique. At the (k)-skeleton for $k \leq n-2$, the complex contains all $(k+1)$-cliques of K_n, and the chromatic number of this skeleton equals the chromatic number of K_n itself, which is n. Since K_n is planar only for $n \leq 4$, for $n \geq 5$ the quotient graph is not the surface graph but a higher-order complex, and the chromatic number grows without bound as n increases. For surface-level quotient graphs (which are planar), the transition to higher-order skeleta introduces non-planar structure as soon as the clique complex contains 5-cliques or larger, and the chromatic number is no longer bounded by the surface geometry. □

Remark 6.2. Proposition 6.1 establishes that the four- and five-color bounds are specific to the observational surface. They do not propagate up the tower. As the system's representational architecture ascends through higher-order skeleta, new chromatic demands arise that are not constrained by the planarity of S^2.

7 Three Levels of the Sensory Count

We now interpret the chromatic structure of the framework against the historical landscape of sensory enumeration.

7.1 The Historical Structure of the Problem

The question of how many senses an observer possesses has three historically distinct answers, corresponding to three periods of analysis.

The *classical enumeration* identifies five: sight, hearing, smell, taste, and touch. This is Aristotle's account in *De Anima*, and it remained the dominant framework in Western thought through the early modern period. The stability of the count across this period is notable; the five senses were not merely a philosophical convenience but a phenomenologically robust enumeration arrived at by reflective attention to the structure of perception.

The *reductionist tradition*, sharpened in the analytic period, has sought to reduce the count. Functionalist analysis notes that some of the classical five are partially decomposable: touch, for instance, involves pressure, temperature, and pain as distinguishable submodalities. On strict individuation criteria, the classical five may compress toward fewer genuinely irreducible modes. This tradition has not produced a stable consensus but has consistently applied pressure in the direction of four or fewer.

The *modern proliferation*, initiated by Sherrington's identification of proprioception in 1906 [2], has pressed in the opposite direction. Contemporary sensory biology recognizes proprioception (body position), the vestibular sense (balance and acceleration), thermoception (temperature), nociception (pain), interoception (internal organ states), and further modes depending on criteria of individuation, yielding counts of twenty or more in current literature. The proliferation has not stabilized; each investigation into sensory architecture tends to individuate new modes.

These three stances — five (stable classical), four or fewer (reductionist), twenty or more (modern biology) — have not been reconciled on empirical grounds. The criteria for what counts as a distinct sense are not fixed by observation alone.

7.2 The Structural Account

The chromatic framework of the present paper resolves the three-way structure without remainder.

Definition 7.1 (Sense as Irreducible Distinguishing Mode)**.** A *sense* at simplicial order k is an irreducible distinguishing resource at that order: a mode of discrimination that cannot be reduced to combinations of other modes at the same order without representational loss. The number of senses at order k is the chromatic number $\chi(X(Q_{w^*})^{(k)})$ of the k-skeleton of the observational clique complex.

At $k = 0$ — the observational surface itself — Theorems 4.3 and 5.2 yield:

Corollary 7.2 (The Three-Level Sensory Account)**.** *For any embedded epistemic system satisfying Assumption 3.1:*

(i) Classical count (five). At most five irreducible distinguishing modes are constructively sufficient at the observational surface: $\chi(Q_{w^*}) \leq 5$. Five is the number arrived at by working through the surface constructively, following the Kempe-chain algorithm.

(ii) Tight bound (four). At most four irreducible distinguishing modes are required at the observational surface, and this bound is tight: $\chi(Q_{w^*}) \leq 4$ with equality achievable. Four is the non-constructive minimum, established only by the Four Color Theorem.

(iii) *Modern proliferation (unbounded). At simplicial order $k \geq 1$ the chromatic number $\chi(X(Q_{w^*})^{(k)})$ is not bounded by the surface geometry. As representational sophistication ascends the tower, new distinguishing modes become individuable at each order, and the count grows without a fixed ceiling.*

Remark 7.3 (The Stability of Five)*.* The Aristotelian count of five is stable because five is the constructive chromatic bound on the observational surface. An enumerator working by reflective attention — proceeding through the perceptual modes available from within the surface and asking which are irreducible — will arrive at five, because five is the number the constructive algorithm requires in the maximal-degree case. The stability of this count across two millennia of philosophical reflection is not accidental; it corresponds to the constructive completeness of the Five Color Theorem at the surface level.

Remark 7.4 (The Reductionist Tradition and the Four-Color Bound)*.* The reductionist tradition's pressure toward four or fewer is likewise not arbitrary. Four is the tight chromatic bound: the minimum number of genuinely irreducible modes required in any surface configuration. The Four Color Theorem establishes that five is never necessary — that the fifth mode can always be eliminated by a suitable recoloring of the surface. Reductionist analysts who have sought to compress the five senses have been tracking the difference between the constructive and tight bounds, without the formal apparatus to state it precisely.

Remark 7.5 (Modern Sensory Biology and the Tower)*.* The upward pressure of modern sensory biology corresponds to ascending the simplicial tower. Sherrington's proprioception, and the subsequent identification of vestibular, thermoceptive, nociceptive, and interoceptive modes, represents the individuation of distinguishing resources at higher simplicial orders — levels of representational structure that are invisible at the surface but become articulate as the system's self-representational capacity increases. Each new mode discovered by sensory biology is a new chromatic demand at some order $k > 0$ of the tower. The count is not bounded above because the tower is not bounded above. This explains why the proliferation has not stabilized: it will continue as long as representational analysis continues to ascend.

Remark 7.6 (Connection to the Reflexivity Condition)*. The Imagination Machine I* establishes the inclusion $C \subseteq D$: classifiers are themselves elements of the observation space. This is the condition that makes a system genuinely epistemic rather than a mere transducer. The passage from the surface to the tower is the structural correlate of this condition. At the surface ($k = 0$), the system discriminates the environment. Ascending to $k = 1$ and beyond, the system begins to discriminate its own discriminations — to individuate modes of perception as objects of representational attention. The proliferation of sensory modes in modern biology is, in these terms, the scientific expression of $C \subseteq D$: as the system's reflexive capacity deepens, it finds more to say about its own observational structure.

8 Discussion

The central result of this paper is that the chromatic number of the observational quotient graph is a derived invariant of the embedding geometry, not an empirical contingency. The two-sphere topology forces planarity; planarity forces the five-color constructive bound by an argument internal to the paper; and the four-color tight bound follows by citation of the Four Color Theorem. The three historically distinct answers to the question of how many senses an embedded observer possesses are jointly explained: five by constructive completeness, four by tightness, and the modern proliferation by the unbounded chromatic structure of the simplicial tower above the surface.

Several questions remain open.

The geometric assumption. Assumption 3.1 is grounded in the hypersphere geometry of *The Imagination Machine XIV*. Whether the two-sphere topology of the observational surface is derivable from more primitive conditions within the framework — in particular from the fixed-point conditions of *The Imagination Machine I* alone, without the geometric picture — remains open. A derivation from the inference–implication loop alone would substantially strengthen the result by removing the geometric assumption as an independent postulate.

Chromatic structure of the tower. Proposition 6.1 establishes that the chromatic number grows without bound in the simplicial tower, but does not characterize the growth rate or the specific chromatic demands at each order for observational quotient complexes. A fuller treatment would give the chromatic number $\chi(X(Q_{w^*})^{(k)})$ as a function of k and the structure of Q_{w^*}, providing a quantitative account of the rate at which new sensory modes become individuable as representational sophistication increases.

Topology on Q_{w^*}. The interpretation of chromatic number as counting irreducible sensory modes requires adjacency in Q_{w^*} to encode observational nearness. This is conceptually natural but requires a precise specification: a topology on the quotient graph that makes adjacency epistemically meaningful. The probability structure (D, Σ_D, μ_D) of *The Imagination Machine I* provides the measure-theoretic materials for this specification, but the explicit construction is left for subsequent work.

The series began with the claim that knowledge is necessarily local — a view from somewhere, never from nowhere. The present paper adds a quantitative dimension to that claim: the locality of the view imposes a precise and finite constraint on the discriminative resources any embedded observer requires at the surface, and an open-ended proliferation of resources as representational depth increases. The geometry of being inside determines, at the surface, exactly how many ways there are to tell things apart — and leaves the deeper structure of discrimination boundlessly open to exploration.

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