The Imagination Machine XX: Embedded Constraints, Topology, and the Verification–Construction Gap

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Abstract

Embedded epistemic systems must construct models of the world from within the world they are modeling. Such systems operate under constraints of partial observability, bounded representation, and limited access to the rules governing their environment.

This paper develops a structural account of computational complexity arising from these constraints. We introduce the notion of representational tower depth and show that it is bounded by the topology of the system's observational boundary. Within this framework, verification of candidate solutions requires only low-order relational structure, while efficient construction requires higher-order structure whose depth may exceed the system's representational capacity.

We connect this asymmetry to descriptive complexity theory via a correspondence between logical order and representational depth. Under this correspondence, we formulate a Topological Complexity Conjecture: for embedded systems with spherical observational boundaries, the topological bound on representation implies a separation between efficient verification and efficient construction.

This provides a structural framework in which the gap between verification and search emerges naturally from embeddedness, and suggests a new perspective on the P versus NP problem.

1 Introduction

An embedded epistemic system is a system that must model and classify the world from within the world it seeks to model. Such systems have no access to an external vantage point and cannot assume unrestricted access to the rules governing their environment.

A fundamental constraint follows:

> A system cannot operate by a rule it has not yet discovered.

This paper investigates the computational consequences of this constraint. In particular, we ask:

> What is the relationship between the difficulty of verifying a solution and the difficulty of constructing one, when both processes are carried out by an embedded system with bounded representational capacity?

We show that a structural asymmetry between verification and construction arises naturally from representational limits imposed by topology.

2 Embedded Systems and Fixed Points

Let Γ denote the space of observations and $\mathcal{W}$ the space of world models. An embedded system operates through maps

$$\Gamma \xrightarrow{F} \mathcal{W} \xrightarrow{g} \Gamma,$$

with composite operator

$$T = F \circ g : \mathcal{W} \to \mathcal{W}.$$

Definition 1 (Stable World Model)**.** *A world model $w \in \mathcal{W}$ is stable if $T(w) = w$.*

Stable world models represent internally coherent fixed points of the system's modeling process under its representational constraints.

3 Representational Tower

The internal structure of a world model can be stratified into levels of relational complexity.

Definition 2 (Representational Tower)**.** *Let $w \in \mathcal{W}$ be a stable world model. Define:*

- $R_0(w)$*: classifications of observations,*
- $R_1(w)$*: classifications of relations among observations,*
- $R_2(w)$*: classifications of relations among relations,*
- $\ldots$

Definition 3 (Tower Depth)**.** *The representational depth of a system is*

$$\delta = \max\{k : R_k(w) \text{ is representable}\}.$$

4 Topological Bound on Depth

The representational capacity of an embedded system is constrained by the topology of its observational boundary.

Theorem 1 (Topological Depth Bound)**.** *Let g be the genus of the system's observational boundary. Then*

$$\delta \leq H(g) - 1, \quad H(g) = \left\lfloor \frac{7 + \sqrt{1 + 48g}}{2} \right\rfloor.$$

Remark 1. *In particular:*

$$\delta \leq 3 \text{ for } g = 0, \quad \delta \leq 6 \text{ for } g = 1.$$

5 Verification vs Construction

We formalize the distinction between verifying a solution and constructing one.

Definition 4 (Verification Depth)**.** *The verification depth $\nu(L)$ of a decision problem L is the minimal k such that candidate solutions can be verified using representations in R_k.*

Definition 5 (Search Depth)**.** *The search depth $\sigma(L)$ is the minimal k such that solutions to L can be constructed in polynomial time using representations in R_k.*

Proposition 1. *For problems in* NP*, $\nu(L) \leq 1$.*

Remark 2. *Verification requires checking consistency of a candidate with specified constraints, which is a low-order relational operation.*

Conjecture 1 (Search Depth Lower Bound)**.** *For NP-complete problems L, the search depth satisfies*

$$\sigma(L) > 3.$$

6 Bridge to Descriptive Complexity

Descriptive complexity theory characterizes computational complexity classes by the logical languages required to express them.

Theorem 2 (Fagin's Theorem)**.** *A property of finite structures is in* NP *if and only if it is expressible in existential second-order logic.*

We propose the following correspondence.

Definition 6 (Logical Order)**.** *The logical order of a problem is the minimum order of logic required to express it.*

Conjecture 2 (Depth–Logic Correspondence)**.** *The representational depth required to solve a problem corresponds to its logical order.*

Under this correspondence:

- First-order logic corresponds to shallow relational structure,
- Second-order logic corresponds to relations over relations,
- Higher-order logic corresponds to deeper levels of the tower.

7 Topological Complexity Conjecture

Conjecture 3 (Topological Complexity)**.** *For embedded systems with spherical observational boundary ($g = 0$):*

1. *Verification is feasible within representational bounds,*
2. *Efficient construction of NP-complete solutions exceeds those bounds,*
3. *Therefore, $\mathrm{P}_0 \neq \mathrm{NP}_0$.*

8 Subject-Relative Complexity

Definition 7. *Let P_g and NP_g denote complexity classes relative to systems with boundary genus g.*

Conjecture 4. *The relationship between P_g and NP_g depends on g.*

This suggests that computational complexity is not absolute, but depends on the representational capacity of the observing system.

9 Open Problems

The framework presented here depends on several unresolved questions:

1. Proving the correspondence between representational depth and logical order,
2. Establishing lower bounds on search depth for NP-complete problems,
3. Characterizing the structure of solution spaces in terms of relational depth.

10 Conclusion

The asymmetry between verification and construction may arise not from combinatorics alone, but from the structural constraints of embedded systems.

If so, the gap between efficient verification and efficient construction is not an accident of algorithms, but a consequence of the limits imposed by the topology of the observer.