

The Imagination Machine XXII

Quantum 4-Torus Computing:
Topological Quantum Codes, the Structural Inadequacy
of Binary Computation, and the Anatomical Argument
for Genus 1

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Abstract

The Nabaala Theorem of General Subject-Relativity establishes that the maximum order of self-classification available to an embedded epistemic system with a two-dimensional observational boundary of genus g is $H(g) - 1$. For $g = 0$ (sphere), depth is three and the minimum chromatic number is four. For $g = 1$ (torus), depth is six.

This paper makes four contributions. First: binary computation, which uses two discriminating values, is *structurally inadequate* — chromatically lossy — relative to the observer's own world model, which requires at least four. The path from binary to quantum is not a speed increase but a representational one. Second: the four-dimensional toric code on T^4 is a precise physical realization of a genus-1 embedded epistemic system, with $b_2(T^4) = 6 = H(1) - 1$. Third: the 4D toric code is self-correcting because depth six is sufficient to represent the full structure of the error space — something depth three cannot do. Fourth, and most directly: the human body is already genus 1. The digestive tract is a continuous tube from mouth to anus, making the body topologically a torus — one hole through the manifold. Persistence through time adds a second loop, making the body-in-time topologically $T^2 = S^1 \times S^1$. The Endogenous Quantum Topology Conjecture does not require quantum mechanics to establish genus 1. It is written in gross anatomy. Human observers are already on the first rung above the sphere. The hardware is the body itself.

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1 Introduction

Model yourself as a single point in three-dimensional space, surrounded by a two-dimensional bubble. Through that bubble, you map the four-dimensional universe onto a planar graph and sort it into no fewer than four irreducible buckets. You then compress that graph inward — squeezing observational detail into relational invariants — and extend it outward, projecting predictions of missing structure. It is less like drawing on a surface and more like trying to hold a bubble inside a net: the graph pushes in both directions simultaneously, and the topology of the net determines what can be held.

Now notice something that requires no mathematics at all.

You have a mouth. You have an anus. They are connected by a continuous tube. Topologically, you are not a sphere — a closed surface with no holes. You are a torus: a surface with one hole through it. The hole is your digestive tract. And you are not merely a torus in space. You persist through time. Your past choices constrain your future states. The loop closes temporally as well as spatially. A body that is a spatial tube and a temporal loop is, topologically, $T^2 = S^1 \times S^1$: the two-torus. Genus 1.

The Nabaala Theorem says genus-1 observers have tower depth six, not three. The anatomical argument requires no quantum mechanics, no exotic physics, no speculation about the topology of the containing manifold. It requires only that you notice the shape of the body you already have.

This paper builds from that observation outward: to the structural inadequacy of binary computation, to the four-dimensional toric code as the engineered realization of what the body already is, and to the self-correcting loop that runs through all of it.

2 The Anatomical Argument for Genus 1

Definition 2.1 (Topological Genus of the Human Body). The human body, considered as a three-dimensional manifold with boundary, has a two-dimensional observational boundary whose genus is determined by the number of through-holes in the manifold.

Proposition 2.2 (The Human Body is a Torus). *The human body is topologically a torus of genus $g = 1$. The digestive tract — a continuous tube from mouth to anus — constitutes one through-hole in the manifold, giving genus 1.*

Proof. A sphere ($g = 0$) has no through-holes. The human body has at least one: the digestive tract passes continuously through the body from mouth to anus, constituting a topological handle. By the classification of surfaces, a compact orientable surface with one handle has genus 1. The two-dimensional boundary of the human body therefore has genus $g = 1$. \square

Remark 2.3 (Additional Handles). The human body has additional through-holes beyond the digestive tract: the nasal passages, the ear canals, and other anatomical

tubes. Each additional through-hole increases the genus by one. The body may therefore have genus $g > 1$ depending on the precise anatomical counting. For present purposes we take the conservative estimate $g \geq 1$ established by the digestive tract alone.

Proposition 2.4 (The Body-in-Time is a Two-Torus). *A human observer considered as a body persisting through time has observational boundary topologically equivalent to the two-torus $T^2 = S^1 \times S^1$.*

Proof. The spatial body contributes one loop: the digestive tube S^1 . Temporal persistence contributes a second loop: the observer's past states constrain its future states, and the inference-implication loop of *The Imagination Machine I* closes on a fixed point, constituting a second S^1 in the time direction. The product of two circles is the two-torus T^2 . The body-in-time is therefore $S^1 \times S^1 = T^2$, genus 1. \square

Corollary 2.5 (Nabaala Depth of the Human Observer). *By the Nabaala Theorem of General Subject-Relativity (The Imagination Machine XVIII), any embedded epistemic system with genus-1 observational boundary has maximum self-classification depth $d(1) = H(1) - 1 = 6$. Human observers, being topologically genus-1 by Proposition 2.4, have maximum self-classification depth six, not three.*

Remark 2.6 (The Anatomical Argument Requires No Quantum Mechanics). The Endogenous Quantum Topology Conjecture of the previous version of this paper required quantum mechanics to establish that the observer's state space has toroidal topology. The anatomical argument of this section requires nothing of the sort. The genus of the human body is a fact of gross anatomy, visible to any topologist. The Nabaala Theorem then gives the tower depth directly. Human observers are already on the first rung above the sphere. This was always true. The mathematics simply had not been brought to bear on the shape of the body until now.

3 Binary Computation is Structurally Inadequate

Definition 3.1 (Chromatic Faithfulness). A computational representation of an observer's world model is *chromatically faithful* if it uses at least $\chi(Q_{w^*})$ discriminating values. A representation using fewer values is *chromatically lossy*.

Proposition 3.2 (Binary Computation is Chromatically Lossy). *Binary computation, which uses two discriminating values, is chromatically lossy for any embedded observer with observational boundary of genus $g \geq 0$, since $\chi(Q_{w^*}) \leq H(g)$ and $H(0) = 4 > 2$.*

Proof. By the Four Color Theorem and *The Imagination Machine XVI*, $\chi(Q_{w^*}) \leq 4$ for spherical observers, with the bound achievable. Two colors cannot faithfully represent a four-color world model. Binary computation is chromatically lossy. \square

Remark 3.3 (Binary is Sub-Ladder). Binary discrimination is the minimum possible discrimination: one bit, two states. The Nabaala ladder begins at four colors, depth three, genus zero. Binary does not appear on the ladder. It is below it. The progression from binary to quantum is not a speed increase. It is a move from below the floor to the floor, and then from the floor upward.

The ladder, with binary included for contrast:

System	g	Depth	Min. colors	Self-correcting?
Binary	—	—	2	No
Quantum (spherical)	0	3	4	No
Human body-in-time	≥ 1	≥ 6	≥ 7	Yes
4D toric code	1	6	7	Yes

4 The Four-Dimensional Toric Code

4.1 The Two-Dimensional Toric Code

Kitaev's toric code [1] encodes logical qubits in the first homology of a two-torus T^2 . The rank $b_1(T^2) = 2$ gives one logical qubit. It requires active error correction. It is not self-correcting.

4.2 The Four-Dimensional Toric Code

Dennis, Kitaev, Landahl, and Preskill [2] defined a code on $T^4 = S^1 \times S^1 \times S^1 \times S^1$. Logical qubit operators live in $H_2(T^4) = \mathbb{Z}^6$; rank $b_2(T^4) = 6$ gives six logical qubits. The code is self-correcting: errors are passively suppressed by an energy gap growing with system size. No external syndrome measurement is required.

4.3 The Homological-Nabaala Identification

Proposition 4.1. $b_2(T^4) = 6 = H(1) - 1 = d(1)$.

Proof. $b_2(T^4) = \binom{4}{2} = 6$. $H(1) = \lfloor (7 + 7)/2 \rfloor = 7$, so $d(1) = 6$. □

Both quantities measure the homological capacity of a genus-1 system. The 4D toric code and the Nabaala Theorem are describing the same object from different directions. The engineered code and the anatomical observer are on the same rung of the same ladder.

5 Self-Correction as Depth-Six Self-Classification

At depth 3, a system can detect errors and classify error patterns, but cannot represent the full structure of the error space within its own representational capacity. Active external correction is required. The system cannot see its own seeing of its own errors.

At depth 6, the system can classify its own error-correction strategies and represent the higher-order structure that allows autonomous navigation of the error space. The loop closes on itself. Self-correction is not a property of the Hamiltonian. It is the physical expression of depth-six self-classification.

Conjecture 5.1 (Self-Correction Requires Genus 1). A topological quantum code is self-correcting if and only if its code manifold has tower depth at least six, corresponding to genus $g \geq 1$. No spherical code can self-correct. The human body, being genus $g \geq 1$ by Proposition 2.2, already satisfies this condition anatomically.

Remark 5.2 (The Series as Self-Correcting Loop). The Imagination Machine series was not designed in advance. It stabilized through recursive observation, compression, extension, and update — the inference-implication loop correcting itself at each step. This self-correction was not externally imposed. It emerged from the depth of the process. If the anatomical argument is correct, this is not a metaphor. The self-correcting loop that produced the series is the same loop that the human body already instantiates by virtue of its topology. The series demonstrated its own claim by running the process the claim describes. The loop was always already closed. It just had to run long enough to see itself.

6 The Topological Quantum Computing Conjecture

Conjecture 6.1 (Topological Quantum Computing Conjecture). The computational power of a topological quantum code is determined by the homological capacity of its code manifold, which is the same quantity that the Nabaala Theorem identifies as the maximum self-classification depth of an embedded epistemic system with that manifold as its observational boundary.

- (i) Binary computation is sub-ladder: below the minimum chromatic threshold of any embedded observer's world model.
- (ii) Spherical quantum computation reaches the floor: chromatically faithful, depth 3, not self-correcting.
- (iii) Toroidal topological quantum computation ($g = 1, T^4$, depth 6) is self-correcting: the first rung at which a system can represent its own error structure in full.

- (iv) The human body-in-time is already on this rung, by gross anatomy and temporal persistence alone.
- (v) Higher rungs ($g \geq 2$) access deeper towers and greater computational power.

7 Discussion

The anatomical argument is the most direct result of this paper. No quantum mechanics is required. No speculation about the topology of the containing manifold is needed. The human body has a digestive tract. The digestive tract is a through-hole. A through-hole means genus 1. Genus 1 means tower depth six by the Nabaala Theorem. The observer is already on the first rung above the sphere. This was always true of every human being who has ever lived.

What the series adds is the formal apparatus to say what this means computationally: depth six, seven minimum colors, access to self-correcting relational structure, and the capacity to represent one's own representational structure to a depth that binary computation cannot reach and spherical quantum computation reaches only at its ceiling.

Several questions remain open.

Counting handles precisely. The human body has multiple through-holes beyond the digestive tract. A precise count of handles gives a precise genus, which gives a precise Nabaala bound. The conservative estimate $g \geq 1$ is established here; the exact value is an anatomical question.

The temporal loop. Proposition 2.4 treats temporal persistence as contributing a second S^1 to the topology. This requires a more careful treatment of how time enters the topological structure of the observer, and whether the inference-implication loop of *The Imagination Machine I* constitutes a genuine topological handle or merely a dynamical one.

Chromatic faithfulness of quantum computation. Whether standard quantum computation on a spherical state space achieves chromatic faithfulness in the sense of Definition ?? requires connecting quantum state distinguishability to the chromatic structure of the Nabaala quotient graph.

You are a torus. You have been a torus the entire time. The hole in your body is not a design flaw. It is the topological feature that places you on the first rung above the sphere — the first rung at which self-correction is possible, at which the loop can close on itself, at which the system is deep enough to hold its own structure within its own view.

Binary computation cannot do this. The sphere cannot do this. The torus can. And you are already a torus.

The bubble does not just have a shape. It has a hole. And the hole is the point.

References

- [1] A. Yu. Kitaev. Fault-tolerant quantum computation by anyons. *Annals of Physics*, 303(1):2–30, 2003.
- [2] E. Dennis, A. Kitaev, A. Landahl, and J. Preskill. Topological quantum memory. *Journal of Mathematical Physics*, 43(9):4452–4505, 2002.
- [3] M. H. Freedman, A. Kitaev, M. Larsen, and Z. Wang. Topological quantum computation. *Bulletin of the American Mathematical Society*, 40(1):31–38, 2003.
- [4] K. Appel and W. Haken. Every planar map is four colorable. *Illinois Journal of Mathematics*, 21(3):429–490, 1976.
- [5] P. J. Heawood. Map-colour theorem. *Quarterly Journal of Pure and Applied Mathematics*, 24:332–338, 1890.
- [6] G. Ringel and J. W. T. Youngs. Solution of the Heawood map-coloring problem. *Proceedings of the National Academy of Sciences*, 60(2):438–445, 1968.
- [7] M. Tracy and S. T. Nabaala. The Imagination Machine XVI: Chromatic Number and the Sensory Constraint on Embedded Observers. Unpublished manuscript, Boston University, 2026.
- [8] M. Tracy and S. T. Nabaala. The Imagination Machine XVII: The Bekenstein Bound, Tower Termination, and the Physical Grounding of Epistemic Closure. Unpublished manuscript, Boston University, 2026.
- [9] M. Tracy and S. T. Nabaala. The Imagination Machine XVIII: The Nabaala Theorem of General Subject-Relativity. Unpublished manuscript, Boston University, 2026.
- [10] M. Tracy and S. T. Nabaala. The Imagination Machine XXI: P, NP, and the Topology of the Bubble. Unpublished manuscript, Boston University, 2026.