The Imagination Machine XXV: Simplicial Completion and Functorial Representation

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March 2026

Abstract

We reformulate the Imagination Machine framework entirely in simplicial and categorical language. Systems that operate under partial information are modeled as constructing simplicial complexes through iterative horn-filling. The dynamics of completion are expressed as functorial mappings between categories of partial and complete structures.

Local constraint imposition is interpreted as the specification of compatible face maps, while global coherence corresponds to the existence of limits or fixed points under completion operators. Distributed systems are modeled as diagrams whose consistency conditions enforce global structure without central representation.

This formalism provides a unified account of learning, representation, and inference as simplicial completion under functorial dynamics.

1 Introduction

The Imagination Machine framework characterizes learning systems as operating on partial relational structure and completing it under constraint. Previous formulations described this process in graph-theoretic, dynamical, and neuroscientific terms.

In this paper, we provide a coordinate-free reformulation in simplicial and categorical language. The goal is not to introduce new structure, but to express the same mechanism in a formalism where compositionality, partiality, and completion are intrinsic.

2 Simplicial Structure

Definition 1 (Simplicial Complex). *A* simplicial complex K *is a collection of simplices closed under the operation of taking faces.*

Definition 2 (Horn). *Let* Δ^n *be the standard* n*-simplex. A* horn Λ^n_k *is the union of all* $(n-1)$*-faces of* Δ^n *except the* k*-th face.*

Definition 3 (Horn Filling). *A* horn filling *is a map extending a horn* $\Lambda^n_k \to K$ *to a full simplex* $\Delta^n \to K$*.*

Remark 1. *Horn filling represents the completion of partial relational structure.*

3 Systems as Simplicial Constructors

Definition 4 (Partial Structure)**.** *A* partial structure *is a simplicial complex containing horns that are not yet filled.*

Definition 5 (Completion Operator)**.** *A* completion operator *is a map*

$$\Phi : K_{partial} \to K_{complete}$$

that fills admissible horns in a simplicial complex.

Proposition 1. *Learning corresponds to the iterative application of horn-filling operations.*

4 Categorical Formulation

Definition 6 (Category of Structures)**.** *Let $\mathcal{C}$ be a category whose objects are simplicial complexes and whose morphisms preserve face relations.*

Definition 7 (Completion Functor)**.** *A* completion functor

$$F : \mathcal{C}_{partial} \to \mathcal{C}_{complete}$$

maps partial structures to completed structures.

Remark 2. *The functor F need not be unique; multiple completions may exist.*

Definition 8 (Fixed Point)**.** *A* fixed point *of F is an object K such that*

$$F(K) \cong K.$$

Proposition 2. *Coherent representations correspond to fixed points of completion functors.*

5 Local Constraints as Face Conditions

Definition 9 (Face Compatibility)**.** *A set of simplices is* compatible *if their face maps agree on overlaps.*

Proposition 3. *Local constraint imposition corresponds to enforcing compatibility of face maps.*

Remark 3. *In this formulation, what appears as a local constraint is a restriction on allowable gluings of simplices.*

6 Distributed Completion as Diagrammatic Consistency

Definition 10 (Diagram)**.** *A* diagram *in $\mathcal{C}$ is a functor $D : J \to \mathcal{C}$ from an index category J.*

Definition 11 (Limit)**.** *A* limit *of a diagram D is a universal object L together with compatible morphisms to each object in the diagram.*

Theorem 1 (Distributed Completion)**.** *Global coherence of a system corresponds to the existence of a limit of the diagram defined by its local structures.*

Proof. Each local structure imposes compatibility conditions. A limit object satisfies all such conditions simultaneously, representing a globally coherent completion. □

Corollary 1. *Global structure is not contained in any single object but emerges as a universal solution to compatibility constraints.*

7 Event-Based Constraints as Local Morphisms

We now reinterpret discrete updates.

Definition 12 (Event Morphism)**.** *An* event morphism *is a morphism that restricts admissible extensions of a partial structure by imposing a local compatibility condition.*

Proposition 4. *Event morphisms correspond to local constraints that refine the space of admissible horn fillings.*

Remark 4. *Discrete updates do not transmit complete information but constrain allowable extensions of the structure.*

8 Manifolds and Coordinate Systems

Definition 13 (Geometric Realization)**.** *The geometric realization $|K|$ of a simplicial complex K is a topological space obtained by gluing simplices.*

Proposition 5. *Low-dimensional manifolds arise as geometric realizations of simplicial complexes with compatible gluing conditions.*

Definition 14 (Coordinate System)**.** *A coordinate system is a functor assigning representations to simplices in a way that preserves relational structure.*

Proposition 6. *Different coordinate systems correspond to functorial representations of the same underlying simplicial complex.*

9 Synthesis

Theorem 2 (Simplicial Completion Principle)**.** *Any system operating under partial information constructs a simplicial complex through iterative horn-filling, with global coherence corresponding to fixed points or limits under completion functors.*

10 Conclusion

We have shown that the Imagination Machine framework admits a natural expression in simplicial and categorical language. Learning, inference, and representation are unified as processes of simplicial completion under functorial dynamics.

This formulation removes dependence on any particular domain and reveals the underlying structure common to all systems that must construct coherence from partial information.