

# The Imagination Machine XXVI: Functorial Equivalence Between Neural Dynamics and Simplicial Completion

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## Abstract

We establish a correspondence between neural computation and simplicial completion by constructing a functorial mapping between the two domains. Neural systems are modeled as networks of locally bounded units generating event-based constraints over time, while simplicial systems are modeled as complexes constructed through horn-filling under compatibility conditions.

We show that spike events correspond to local morphisms imposing face constraints, neural dynamics correspond to iterative completion operators, and stable representations correspond to fixed points or limits. Under this mapping, allocentric and egocentric representations correspond to functorial coordinate transformations on a shared underlying structure.

This establishes that neural dynamics and simplicial completion are not merely analogous, but structurally equivalent under a change of representation.

## 1 Introduction

Previous papers in the Imagination Machine series developed two parallel descriptions of learning systems:

- A *neural/dynamical description*, in which systems operate through event-based updates, distributed constraints, and attractor dynamics.
- A *simplicial/categorical description*, in which systems construct simplicial complexes through horn-filling and enforce global coherence via categorical limits.

The present paper establishes a formal correspondence between these two descriptions.

## 2 Categories of Neural and Simplicial Systems

**Definition 1** (Neural Category  $\mathcal{N}$ ). *Objects in  $\mathcal{N}$  are neural systems consisting of:*

- a *network of nodes*,
- a *global state space  $X$* ,
- *event-based update rules*.

*Morphisms in  $\mathcal{N}$  are structure-preserving maps between such systems.*

**Definition 2** (Simplicial Category  $\mathcal{S}$ ). *Objects in  $\mathcal{S}$  are simplicial complexes. Morphisms preserve face relations.*

### 3 Construction of the Functor

**Definition 3** (Functor  $F : \mathcal{N} \rightarrow \mathcal{S}$ ). We define a functor  $F$  mapping neural systems to simplicial complexes as follows:

- Nodes  $\mapsto$  vertices
- Joint activity patterns  $\mapsto$  simplices
- Spike events  $\mapsto$  morphisms imposing face constraints
- Stable states  $\mapsto$  completed simplices

**Remark 1.** This mapping encodes the idea that neural activity constructs relational structure through constraint imposition.

### 4 Correspondence of Operations

**Proposition 1.** Neural event updates correspond to horn-filling operations in simplicial complexes.

*Proof.* A spike event imposes a constraint that restricts allowable configurations of the global state. In simplicial terms, this corresponds to specifying missing faces of a horn, thereby completing it into a simplex.  $\square$

**Proposition 2.** Neural attractor states correspond to fixed points of completion operators.

**Proposition 3.** Distributed neural computation corresponds to diagrammatic completion and limit formation.

### 5 Inverse Mapping

**Definition 4** (Functor  $G : \mathcal{S} \rightarrow \mathcal{N}$ ). We define a functor mapping simplicial complexes to neural realizations:

- simplices  $\mapsto$  coordinated neural activations
- face relations  $\mapsto$  compatibility constraints
- horn-filling  $\mapsto$  dynamic completion processes

**Remark 2.**  $G$  interprets abstract structure as realizable dynamics.

### 6 Equivalence

**Theorem 1.** The functors  $F$  and  $G$  establish a structural equivalence between neural dynamics and simplicial completion up to representation.

*Proof.* Both systems operate on partial structure, impose local constraints, and converge to globally coherent states. The mappings  $F$  and  $G$  preserve these operations and their composition. Therefore, the two systems are equivalent up to change of representation.  $\square$

**Corollary 1.** Neural and simplicial descriptions are coordinate systems on a shared underlying structure.

## 7 Coordinate Transformations

**Proposition 4.** *Allocentric and egocentric representations correspond to functorial changes of coordinates on the underlying structure.*

## 8 Synthesis

**Theorem 2** (Unified Representation Principle). *Learning systems construct coherent representations by imposing local constraints and completing partial structure, regardless of whether they are described in neural or simplicial terms.*

## 9 Conclusion

We have shown that neural dynamics and simplicial completion are functorially equivalent descriptions of the same underlying process. This provides a unified framework in which biological systems, abstract structures, and epistemic processes can be understood as instances of distributed completion under constraint.