The Imagination Machine XXVIII: Black Holes as Topological Gates and the Cosmological Horn-Filling

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March 2026

Abstract

We extend the Imagination Machine framework to cosmological scale. The Big Bang is interpreted as the four-dimensional origin of a bubble expanding through a higher-dimensional constraining topology — the spherical horn-filling one dimension up. Black holes are reinterpreted as topological gates: points of geodesic incompleteness where the three-dimensional fabric of spacetime is pinched, constraining what passes through and shaping its output according to the topological capacity of the boundary. The no-hair theorem is derived as a corollary of the topological capacity of the event horizon S^2: the boundary topology constrains the isometry group of the exterior spacetime to $\mathbb{R} \times U(1)$, which by Noether's theorem yields exactly three conserved quantities — mass M, charge Q, and angular momentum J — exhausting the solution space of the Einstein-Maxwell equations. The gate preserves only what the topology of its boundary can encode. The Bekenstein-Hawking entropy establishes that information is encoded on a boundary of codimension one relative to the constrained region, consistent with the framework's compression principle across all scales. The holographic principle is reinterpreted not as a duality between two theories but as the non-duality between a system and its center: the boundary does not represent the interior — it generates it. Under this interpretation, the embedded observer is not inside the universe looking out; the embedded observer is the boundary condition from which the universe is generated. All results are derived from established theorems of general relativity, quantum field theory, and mathematical physics.

1 Introduction

The Imagination Machine framework establishes that any embedded epistemic system operates on partial relational structure and completes it under constraint. The central mechanism — horn-filling — holds that a system expanding through a constraining topology takes the shape of the last hole it passed through. Previous papers established this principle at the levels of individual cognition (TIM I), neural dynamics (TIM XXIV), simplicial completion (TIM XXV), and functorial equivalence (TIM XXVI).

The present paper asks whether the same principle operates at cosmological scale. We argue that it does, and that the argument requires no new physics — only a reinterpretation of established results. The theorems of Penrose and Hawking, the no-hair theorem, Noether's theorem, the Bekenstein-Hawking entropy formula, and the holographic principle collectively instantiate the horn-filling mechanism at the scale of spacetime itself.

The central new result is Proposition 4.3: the no-hair theorem is derived as a corollary of the topological capacity of the event horizon S^2, via the chain

$$\begin{aligned}&\text{Topology of } S^2\\ &\quad\implies \text{Isometry group } \mathbb{R}\times U(1)\\ &\quad\implies \text{Noether conserved quantities } (M,Q,J)\\ &\quad\implies \text{No-hair theorem.}\end{aligned}$$

This establishes that the gate preserves only what the topology of its boundary can encode — not as a reinterpretation but as a derivation.

We proceed as follows. Section 2 establishes the physical pillars on which the argument rests. Section 3 develops the cosmological horn-filling interpretation. Section 4 addresses black holes as topological gates. Section 5 reinterprets the holographic principle as non-duality. Section 6 connects the cosmological picture to the embedded observer of earlier papers. Section 7 states the unified theorem.

2 Physical Foundations

2.1 The Penrose-Hawking Singularity Theorems

The singularity theorems of Penrose [13] and Hawking [3] establish that under reasonable energy conditions, spacetime geodesics are incomplete: they terminate at finite affine parameter.

Definition 2.1. A spacetime (M, g) is *geodesically incomplete* if there exists a geodesic that cannot be extended to arbitrary affine parameter values.

Theorem 2.2 (Penrose-Hawking [13, 3])**.** *Under the strong energy condition and global hyperbolicity, any spacetime containing a trapped surface is geodesically incomplete.*

Remark 2.3. In the language of the framework, geodesic incompleteness means that the cosmological net has holes. The singularity theorems establish their existence rigorously and unconditionally under physically reasonable assumptions.

2.2 Noether's Theorem

Noether's theorem [11] establishes the foundational connection between symmetry and conservation that underlies the derivation in Section 4.

Theorem 2.4 (Noether [11])**.** *For every continuous symmetry of the action of a physical system, there exists a corresponding conserved quantity. The conserved quantities are in bijective correspondence with the generators of the symmetry group.*

Remark 2.5. This theorem is the bridge between topology and conservation. The isometry group of a spacetime — which is determined by its boundary topology — fixes the number and type of conserved quantities via Noether's theorem. The topological capacity of a boundary is therefore equivalent, through Noether's theorem, to the set of quantities that can be conserved across that boundary.

2.3 The No-Hair Theorem

The no-hair theorem [7, 2, 4] establishes that a stationary black hole solution to the Einstein-Maxwell equations is completely characterized by exactly three parameters: mass M, charge Q, and angular momentum J.

Definition 2.6. The *topological capacity* of a boundary $\partial\mathcal{R}$ is the set of conserved quantities that the topology of $\partial\mathcal{R}$ can encode, as determined by the isometry group of the ambient spacetime via Noether's theorem.

Remark 2.7. The event horizon of a stationary black hole is homeomorphic to S^2 [4, 6]. The topology of S^2, together with asymptotic flatness, constrains the isometry group of the exterior spacetime to $\mathbb{R} \times U(1)$. By Noether's theorem, this group yields exactly two gravitational conserved quantities: mass M from time translation symmetry, and angular momentum J from axial rotational symmetry. The $U(1)$ gauge symmetry of electromagnetism yields a third: charge Q.

The three parameters of the no-hair theorem are not chosen because they are more fundamental than other properties of the infalling matter. They survive because they are what the topology of S^2 can carry via its isometry group. Every other property of the infalling matter — its chemical composition, molecular structure, historical configuration — requires a topologically richer boundary to encode, one whose isometry group has further generators. The event horizon does not have that topology. Therefore those properties are not preserved. The gate encodes only what the topology of its boundary permits.

2.4 Bekenstein-Hawking Entropy

Bekenstein [1] and Hawking [5] established that the entropy of a black hole is proportional to the area of its event horizon:

$$S = \frac{A}{4\ell_P^2}$$

where A is the horizon area and ℓ_P is the Planck length.

Remark 2.8. The constraining topology — the boundary — is always of codimension one relative to the constrained region. The entropy is proportional to area rather than volume because area is the measure of topological capacity: a larger horizon supports a richer isometry group and therefore encodes more conserved quantities. The formula reflects the direct proportionality between boundary topology and information capacity, consistent with the framework's compression principle across all scales.

2.5 The Holographic Principle

The holographic principle, developed by 't Hooft [8] and Susskind [14] and given precise form in the AdS/CFT correspondence of Maldacena [10], states that the maximum information content of a region of space is proportional to its boundary area, not its volume.

Remark 2.9. Standard presentations describe this as a duality between two theories. We argue in Section 5 that this framing is imprecise in a way that matters. The holographic principle establishes not a duality but a non-duality: the boundary does not represent the interior — it generates it.

2.6 The CPT Theorem

The CPT theorem [9, 12] establishes that any Lorentz-invariant quantum field theory is symmetric under the combined operation of charge conjugation (C), parity reversal (P), and time reversal (T).

Remark 2.10. This symmetry connects the Big Bang — a point of outward expansion — with a black hole singularity — a point of inward compression — as CPT reflections of each other. Compression and extension are the same operation run in opposite directions. This was established at the cognitive level in TIM I through the inference-implication loop $T = F \circ g$. The CPT theorem establishes it at the level of fundamental physics. The two results are structurally identical.

3 The Big Bang as Cosmological Horn-Filling

Definition 3.1. The *cosmological bubble* is the observable universe, modeled as a 3-sphere S^3 expanding outward from a compressed origin point.

Definition 3.2. The *cosmological net* is the higher-dimensional constraining topology through which the bubble expands — the four-dimensional structure whose local geometry determines the curvature of spacetime via the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Proposition 3.3. *The Big Bang is the compressed origin of the cosmological bubble — the point to which the bubble can be continuously contracted, corresponding to the limit of maximal compression in the framework.*

Proof. The observable universe is geodesically complete in the forward direction and geodesically incomplete in the backward direction. All backward-directed geodesics terminate at the past singularity by the Penrose-Hawking theorems. This is the limit of maximal compression: the point from which the bubble has been expanding through the cosmological net since the origin. □ □

Proposition 3.4. *The topology of spacetime — encoded in the metric tensor $g_{\mu\nu}$ — determines what trajectories are possible for embedded observers. The geometry was set at the origin.*

Remark 3.5. This is the cosmological version of the toroidal boundary condition established in TIM XXIV: the topology of the space determines what orbits are possible and how. For planets, General Relativity determines the orbit. For embedded observers within a social system, the founding conditions set the geometry. The principle is scale-invariant.

4 Black Holes as Topological Gates

Definition 4.1. A *topological gate* is a region of geodesic incompleteness where the local topology of spacetime constrains the admissible extensions of incoming geodesics and shapes the invariants of whatever passes through according to the topological capacity of its boundary.

Proposition 4.2. *Black holes are topological gates in the sense of the preceding definition.*

Proof. Geodesic incompleteness follows from the Penrose-Hawking singularity theorems. Constraint of output to topological capacity follows from Proposition 4.3 below. □ □

Proposition 4.3. *The no-hair theorem is the physical statement of horn-filling: the output of a topological gate is constrained to the topological capacity of its boundary. Specifically, the topology of the event horizon S^2 determines via Noether's theorem exactly three conserved quantities, which exhaust the solution space of the Einstein-Maxwell equations.*

Proof. A horn filling $\Lambda_k^n \to K$ extends a partial structure to a complete simplex $\Delta^n \to K$. The shape of the completion is determined by the face structure of the horn — the topology of the opening. We establish that the same logic governs the black hole case through the following chain.

The event horizon of a stationary, asymptotically flat black hole is homeomorphic to S^2 [4, 6]. The topology of S^2, together with the requirement of asymptotic flatness, constrains the isometry group of the exterior spacetime to $\mathbb{R} \times U(1)$: time translation symmetry and axial rotational symmetry. By Noether's theorem [11], each continuous symmetry of the action yields exactly one conserved quantity. The two generators of $\mathbb{R} \times U(1)$ yield mass M and angular momentum J. The $U(1)$ gauge symmetry of the electromagnetic field yields charge Q.

The isometry group $\mathbb{R} \times U(1)$ has no further generators under the topology of S^2 and asymptotic flatness. Therefore no further conserved quantities exist. The uniqueness theorems of Israel [7], Carter [2], and Hawking [4, 6] establish that the solution space of the Einstein-Maxwell equations consistent with these constraints is exhausted by the Kerr-Newman family, parameterized by (M, Q, J) alone.

The topology of the boundary therefore determines the topological capacity of the gate via the chain:

$$\begin{aligned}
&\text{Topology of } S^2 \\
&\quad \Longrightarrow \text{ Isometry group } \mathbb{R} \times U(1) \\
&\quad \Longrightarrow \text{ Noether conserved quantities } (M, Q, J) \\
&\quad \Longrightarrow \text{ Kerr-Newman family.}
\end{aligned}$$

The gate preserves only what the topology of its boundary can encode, and nothing more. □ □

Theorem 4.4 (Horn-Filling at Cosmological Scale)**.** *Whatever passes through a black hole emerges — as an exterior gravitational field — characterized by exactly the invariants (M, Q, J) that the topology of the event horizon S^2 can encode via its isometry group and Noether's theorem. This is the no-hair theorem derived as a horn-filling result: the gate constrains the output to the topological capacity of its boundary, and nothing more.*

Remark 4.5. The Hawking radiation spectrum extends this result. The black hole radiates thermally with temperature:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

The thermal spectrum is the maximally uncertain distribution consistent with the known invariants (M, Q, J) — precisely the maximally conservative completion of the partial structure, breaking no additional symmetry. This is the physical analogue of the result established for the embedded observer in TIM VIII: the maximally conservative completion of partial structure is the one that breaks no symmetry beyond what the constraints require.

4.1 Geodesics as Paths of Least Topological Resistance

In General Relativity, free particles follow geodesics — the curves of extremal proper time in curved spacetime, determined entirely by the metric.

Proposition 4.6. *Free particle trajectories near black holes are geodesics following the path of least topological resistance toward the next admissible opening in the spacetime manifold.*

Remark 4.7. This is not a new physical claim. It is the standard geodesic equation reinterpreted in the framework's language. What the framework adds is the recognition that geodesic incompleteness is not merely a technical pathology but a structural feature: it is the physical signature of a topological gate. The geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

encodes the constraint that the path follow the curvature of the net. The Christoffel symbols $\Gamma^\mu_{\nu\rho}$ are the local expression of the net's topology.

5 The Holographic Principle as Non-Duality

Standard presentations of the holographic principle describe it as a duality between a gravitational theory in a bulk region and a quantum field theory on its boundary. We argue this framing is imprecise in a way that matters for the framework.

Definition 5.1. The holographic principle establishes the *non-duality* between a system and its center: the boundary does not represent the interior — it generates it. There is no separate interior that the boundary merely describes.

Proposition 5.2. *The holographic principle is the physical statement of the embeddedness condition: an epistemic system cannot step outside the system it models, because the system and its boundary encoding are not two things in correspondence but one thing expressed at different scales.*

Proof. In AdS/CFT, the boundary theory and the bulk theory are related by a change of variables, not by a correspondence between ontologically separate objects. The boundary is not a map of the interior. The boundary is the interior, encoded on a surface of codimension one. The word "duality" implies two separate things in correspondence. The holographic principle establishes one thing: the system and its center are non-dual. The boundary encodes the interior completely, and the interior is fully determined by the boundary. They are not two theories. They are one structure expressed at two scales. □ □

Remark 5.3. This closes the loop between TIM I and TIM XXVIII. TIM I established the embeddedness condition as the foundational constraint on any epistemic system: an observer cannot step outside the system it models. The holographic principle establishes the same constraint as a theorem of quantum gravity. The embedded observer is not inside the universe looking out. The embedded observer is the boundary condition from which the universe is generated. There is no view from nowhere.

6 Connection to the Embedded Observer

The cosmological picture connects to the embedded observer of earlier papers through the toroidal boundary condition established in TIM XXIV.

The embedded observer is a locally bounded system: a genus-0 body embedded in a toroidal brain-body network. The center of mass is the irreducible point — the compressed origin. The neural net is the constraining topology through which the bubble expands. The tube — mouth to anus — is the topological gate through which the observer acts on and is acted on by the world. The output of that gate is shaped by its topology: circular openings produce lobes, as in the p-orbital, not stacked triangles as in the spherical net. The topological capacity of the opening determines what can emerge.

Proposition 6.1. *The cosmological structure and the cognitive structure are instances of the same topological principle at different scales: compressed origin, constraining net, expansion through the gate, output shaped by the topological capacity of the opening.*

Remark 6.2. The CPT theorem makes this precise. The Big Bang and a black hole singularity are CPT reflections of each other. The cognitive compression-extension cycle — established in TIM I as $T = F \circ g$ — is the same operation at the scale of individual cognition. The universe and the mind operate by the same principle, at every scale at which we have examined it.

7 The Unified Theorem

Theorem 7.1 (Cosmological Horn-Filling Principle)**.** *Any system expanding through a constrained topology takes the shape of the last hole it passed through. The output is constrained to the topological capacity of the boundary through which it passes, as determined by the isometry group of the ambient space via Noether's theorem. This principle holds at every scale at which it has been examined:*

- *Individual cognition: the inference-implication loop $T = F \circ g$ (TIM I)*
- *Neural dynamics: spike events as local constraints on global state (TIM XXIV)*
- *Simplicial completion: horn-filling under functorial dynamics (TIM XXV–XXVI)*
- *Cosmological spacetime: geodesic incompleteness, the no-hair theorem derived via Noether's theorem, and the holographic principle (TIM XXVIII)*

The physical foundations — the Penrose-Hawking singularity theorems, Noether's theorem, the no-hair theorem, the Bekenstein-Hawking entropy formula, the holographic principle, and the CPT theorem — collectively instantiate the mechanism at the scale of spacetime itself, requiring no new physics beyond established results.

Corollary 7.2. *The embedded observer is not inside the universe looking out. The embedded observer is the boundary condition from which the universe is generated. The non-duality between system and center holds at every scale. There is no view from nowhere.*

8 Conclusion

We have shown that the horn-filling principle of the Imagination Machine framework operates at cosmological scale, derivable from established results in general relativity and quantum field theory. The Big Bang is the compressed origin. Black holes are topological gates whose output is constrained to the topological capacity of their boundaries, derived via the chain from boundary topology through isometry group through Noether's theorem to the Kerr-Newman family. The holographic principle is the non-duality between a system and its center.

The universe and the mind operate by the same principle. The framework requires no modification to accommodate this — only recognition that what was established for embedded epistemic systems was always a statement about the structure of reality at every scale at which structure exists.

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