The Imagination Machine XXXIII:
The Cosmic Web as Topological Graph:
A Relational Account of Dark Matter and Dark Energy

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Abstract

The Imagination Machine series established that embedded epistemic systems compress observations into quotient graphs drawn on their observational boundaries, and that the horn-filling principle operates at every scale examined: individual cognition, neural dynamics, simplicial completion, cosmological spacetime, the foundations of mathematics, and quantum field theory. The present paper extends the framework one dimension up. We propose that the cosmic web — the observed large-scale structure of the universe, consisting of filaments, nodes, and voids — is the graph drawn on the three-dimensional manifold of spacetime, precisely one dimension above the two-sphere observational surface of the embedded observer. Black holes occupy the nodes of this graph; filaments are its edges; voids are the bubbles between.

Within this framework, two phenomena that have resisted conventional physical explanation admit natural topological interpretations. Dark matter is reinterpreted as the relational edge structure of the cosmic graph: the topological connective tissue whose gravitational signature is measurable at every scale but whose content is not encodable as local particle matter on the observer's two-sphere boundary. It is not a substance. It is the net itself, casting a gravitational shadow. Dark energy is reinterpreted as the horn-filling dynamics of the cosmological bubble expanding through the constraining topology of the nodal network: the natural consequence of a bubble passing through a constrained nodal structure produces accelerating separation between nodes as the bubble expands, without requiring a new substance or a fine-tuned cosmological constant.

Both phenomena are proposed as structural consequences of the graph topology of spacetime, not independent substances requiring new physics. The cosmic web is Indra's net. The geometry was always already there.

1 Introduction

The Imagination Machine series began with a single constraint: an embedded epistemic system can at most classify the ways in which it classifies the world, within the world itself. It proceeded through formal epistemology, dynamical systems, predictive learning, institutional transmission, analogy, simplicial completion, ethics, geometric theology, categorical formulation, graph theory, computational architecture, philosophy of science, the view from nowhere, chromatic constraints on the senses, the Bekenstein bound, the Nabaala Theorem of General Subject-Relativity, the periodic table, the Closing Loop Theorem, computational complexity, quantum 4-torus computing, the Tracy-Nabaala Theorem, neural dynamics, simplicial equivalences, redundant representational closure, black

holes as topological gates, the Forde-Tracy Theorem on political topology, the Axiom of Choice as a choice of axiom, and the topological incompleteness of quantum field theory.

At every scale examined, the same principle held: any system expanding through a constrained topology takes the shape of the last hole it passed through. The output is characterized by exactly the invariants the topology of the boundary can encode.

The present paper asks whether this principle, already established from individual cognition to cosmological spacetime, extends to the large-scale structure of the universe itself — to the cosmic web of filaments, nodes, and voids that constitutes the observable architecture of matter on scales of hundreds of millions of light years.

We argue that it does. The cosmic web is not merely analogous to the quotient graph of an embedded epistemic system. It is the graph drawn on the three-dimensional manifold of spacetime, one dimension up from the two-sphere observational surface of the embedded observer. The same topological principles that govern the observer's representational architecture govern the architecture of the universe the observer inhabits.

The implications for dark matter and dark energy are not speculative additions to an otherwise complete picture. They follow from the topology.

2 Background: The Horn-Filling Principle at Cosmological Scale

We recall the relevant results from The Imagination Machine XXVIII.

Definition 2.1. A *topological gate* is a region of geodesic incompleteness where the local topology of spacetime constrains the admissible extensions of incoming geodesics and shapes the invariants of whatever passes through according to the topological capacity of its boundary.

Theorem 2.2 (TIM XXVIII, Cosmological Horn-Filling Principle)**.** *Any system expanding through a constrained topology takes the shape of the last hole it passed through. The output is constrained to the topological capacity of the boundary through which it passes, as determined by the isometry group of the ambient space via Noether's theorem. This principle holds at every scale at which it has been examined.*

Proposition 2.3 (TIM XXVIII)**.** *Black holes are topological gates in the sense of Definition 2.1. The no-hair theorem is derived as a corollary: the topology of the event horizon S^2 constrains the isometry group of the exterior spacetime to $\mathbb{R} \times U(1)$, yielding exactly three conserved quantities — mass M, charge Q, and angular momentum J — via Noether's theorem.*

The cosmic web extends this picture. Individual black holes are topological gates at stellar and galactic scale. The cosmic web is the graph formed by the network of such gates at cosmological scale.

3 The Cosmic Web as Graph on the Three-Dimensional Manifold

Definition 3.1. The *cosmic graph* $\mathcal{G} = (V, E, F)$ is defined as follows:

- V is the set of nodes: the massive concentrations of matter at the intersections of cosmic filaments, anchored by supermassive black holes.

- E is the set of edges: the cosmic filaments connecting nodes, along which matter and energy flow and within which dark matter halos are concentrated.
- F is the set of faces: the cosmic voids, the bubbles of relative emptiness enclosed by the filamentary network.

Proposition 3.2. *The cosmic graph $\mathcal{G}$ is drawn on the three-dimensional spatial manifold of the universe, in exact structural analogy with the quotient graph Q_{w^*} drawn on the two-sphere observational boundary of an embedded epistemic system.*

Proof. The Imagination Machine series established that the quotient graph Q_{w^*} is drawn on the observational boundary S^2 of the embedded observer. This boundary is two-dimensional; the graph is a compression of the relational structure of observations onto that surface.

The cosmic web occupies the analogous position one dimension up. The three-dimensional spatial manifold of the universe is the surface on which the cosmic graph is drawn. The nodes, edges, and faces of the cosmic graph are the compressed relational structure of matter and energy at cosmological scale. The embedding is not metaphorical: the filamentary network is the literal graph structure of the spatial manifold, as observed in galaxy surveys and N-body simulations.

The structural analogy is therefore exact: Q_{w^*} is to S^2 as $\mathcal{G}$ is to the three-dimensional spatial manifold. Both are graphs drawn on their respective manifolds; both encode relational structure through compression; both are shaped by the topology of the manifold on which they are drawn. □

□

Remark 3.3. The cosmic graph $\mathcal{G}$ is the graph drawn on the manifold from which the embedded observer's S^2 boundary is itself derived. The observer is inside one of the faces — one of the voids — of the cosmic graph. The observational boundary S^2 is a local cross-section of the larger structure. The cosmic web is the graph one dimension up from the graph on the bubble.

4   Dark Matter as Relational Edge Structure

The standard account of dark matter posits an as-yet undetected particle or class of particles that contributes approximately five times as much mass as ordinary baryonic matter, does not interact electromagnetically, and is detectable only through its gravitational effects. Despite decades of direct detection experiments, no dark matter particle has been identified.

We propose a reinterpretation.

Definition 4.1. The *relational edge structure* of the cosmic graph $\mathcal{G}$ is the set of edges E together with the gravitational field they generate: the topological connective tissue of the cosmic web, whose existence is required by the nodal structure and whose gravitational signature is measurable at every scale.

Proposition 4.2 (Dark Matter as Edge Structure)**.** *What is observed as dark matter is the gravitational signature of the relational edge structure of the cosmic graph $\mathcal{G}$: the topological connective tissue whose content is not encodable as local particle matter on the observer's two-sphere boundary but whose gravitational effects are fully present and measurable.*

Argument. The observational evidence for dark matter is entirely gravitational: anomalous rotation curves in galaxies, gravitational lensing, the large-scale structure of the cosmic web, and the acoustic oscillations in the cosmic microwave background. No non-gravitational signature has been detected.

The Imagination Machine framework establishes that the embedded observer's two-sphere boundary can only encode the relational invariants that survive compression through that boundary. Local particle content — matter at a point — is encodable on S^2 because it is a zero-dimensional object in the three-dimensional manifold, projectable onto the two-dimensional boundary.

The edge structure of the cosmic graph, by contrast, is one-dimensional: it consists of filamentary connections between nodes, extended through the three-dimensional manifold. A one-dimensional object in a three-dimensional manifold is not fully encodable on a two-dimensional boundary: it cannot be compressed to a point without losing its relational character. The edge structure therefore manifests on the boundary only through its integrated gravitational effect — precisely the signature that dark matter detectors observe.

Dark matter is not a substance awaiting detection. It is the net itself: the relational edge structure of the cosmic graph, gravitationally present at every scale but not reducible to local particle content on the observer's boundary. □ □

Remark 4.3. This reinterpretation explains several otherwise puzzling features of dark matter. The absence of a dark matter signal in direct detection experiments follows immediately: the edge structure is not a local particle and therefore cannot produce a local collision signal. The correlation of dark matter halos with cosmic filaments follows from the identification of dark matter with the edge structure of those filaments. The fact that dark matter does not interact electromagnetically follows from its topological rather than particle character: the electromagnetic interaction is a local gauge symmetry, and the edge structure of the cosmic graph is a non-local topological object.

Remark 4.4. Indra's net is a precise description of this structure. Each jewel is a node — a massive concentration of matter anchored by a black hole. Each thread connecting jewels is an edge — a cosmic filament. Each jewel reflects all others because the gravitational field of each node is shaped by the entire nodal network: the edge structure transmits the gravitational influence of every node to every other. The net is not a metaphor. It is the cosmic graph.

5   Dark Energy as Horn-Filling Dynamics

The standard account of dark energy posits a mysterious energy of the vacuum — associated with the cosmological constant Λ in Einstein's field equations — that drives the accelerating expansion of the universe. Its origin and magnitude remain unexplained; the cosmological constant problem is one of the deepest unsolved problems in theoretical physics.

We propose a reinterpretation.

Definition 5.1. The *cosmological bubble* is the observable universe, modeled as expanding outward from its compressed origin through the constraining topology of the cosmic graph $\mathcal{G}$.

Proposition 5.2 (Dark Energy as Horn-Filling Dynamics)**.** *What is observed as dark energy — the accelerating expansion of the universe — is the horn-filling dynamics of the cosmological bubble expanding through the constraining topology of the cosmic graph: the natural consequence of a bubble passing through a nodal network in which nodes recede from one another as the bubble expands through the spaces between them.*

Argument. The horn-filling principle established in TIM XXVIII states that any system expanding through a constrained topology takes the shape of the last hole it passed through. The cosmological

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bubble is expanding through the cosmic graph $\mathcal{G}$, whose nodes are connected by filamentary edges and whose faces are voids.

As the bubble expands through the void faces of the graph, the nodes — the massive concentrations of matter anchored by black holes — recede from one another not because a new repulsive force is pushing them apart, but because the expansion of the bubble through the void structure naturally separates the nodes that bound each void. The voids expand; the nodes at their boundaries move apart.

At early times, when the bubble was compact and the nodes close together, gravitational attraction dominated: the nodes pulled the expansion toward them. At late times, when the bubble has expanded to a scale at which the void faces dominate the topology — when the bubble is passing primarily through void structure rather than nodal structure — the horn-filling dynamics produce accelerating separation between nodes, because the topology of the void is expanding and carrying the nodes with it.

The transition from decelerating to accelerating expansion corresponds to the transition from node-dominated to void-dominated topology in the path of the expanding bubble. This is not a fine-tuning problem. It is a topological transition. □ □

Remark 5.3. The cosmological constant Λ already appears in the Einstein field equations that source the $k = +1$ FRW geometry of the containing manifold. In the standard account, Λ is a free parameter requiring fine-tuning to its observed value. In the present account, Λ is the integrated effect of the horn-filling dynamics of the cosmological bubble passing through the cosmic graph. Its value is determined by the topology of the graph, not by a free parameter. The cosmological constant problem is reframed as a topological question: what is the graph structure of the cosmic web, and what horn-filling dynamics does it produce?

Remark 5.4. The CPT theorem, invoked in TIM XXVIII, established that the Big Bang and a black hole singularity are CPT reflections of each other: compression and extension are the same operation run in opposite directions. The cosmic graph makes this precise: the Big Bang is the compressed origin of the bubble; the black holes are the nodes through which the bubble passes as it expands; the horn-filling dynamics of the expansion are the extension phase of the cosmological compression-extension cycle. The universe is running the inference-implication loop at cosmological scale.

6 The Dimensional Ladder and the Observational Limit

The Nabaala Theorem of General Subject-Relativity established that the maximum order of self-classification available to an embedded epistemic system is a topological invariant of its observational boundary. For a two-sphere boundary, the maximum depth is $H(0) - 1 = 3$.

The present paper proposes that the cosmic graph is one dimension up from the observer's two-sphere boundary. This dimensional relationship is not coincidental.

Proposition 6.1. *The embedded observer cannot directly observe the edge structure of the cosmic graph as particle content for the same reason the observer cannot access the fourth dimension of S^3: both are one dimension above the observer's encodable boundary.*

Proof. The observer's boundary is S^2, a two-dimensional surface embedded in a three-dimensional manifold. The observer can encode zero-dimensional objects (particles at points) and two-dimensional

structures (surfaces, fields) on this boundary, but cannot encode one-dimensional objects (filaments, edges) without reducing them to their endpoints or their integrated effects.

The edge structure of the cosmic graph is one-dimensional in the three-dimensional spatial manifold. It is therefore exactly one dimension above what the observer's S^2 boundary can encode as local content. The observer detects it only through its integrated gravitational effect — precisely as the embedded observer in S^3 detects the fourth dimension only through its geometric consequences, never directly.

The dark matter detection problem is the cosmological instance of the Nabaala observational limit: the observer cannot directly encode what is one dimension above its boundary. $\quad\square\qquad\square$

Remark 6.2. This result connects the dark matter problem to the view from nowhere. The center of S^3 is inaccessible from within the manifold — it is one dimension above the observer's boundary. The edge structure of the cosmic graph is inaccessible as local particle content — it is one dimension above the observer's two-sphere surface. Both are the same structural fact at different scales: the observer cannot directly access what is one dimension above its encodable boundary. The view from nowhere and dark matter are the same non-observation at different scales.

7 Indra's Net as Precise Geometric Description

The Buddhist philosophical concept of Indra's net describes an infinite array of jewels, each reflecting all the others, each reflection containing reflections of reflections, without terminus and without a privileged center. We now establish that this is not a metaphor but a precise geometric description of the cosmic graph.

Proposition 7.1. *Indra's net is a precise geometric description of the cosmic graph $\mathcal{G}$ under the Koopman linearization of the gravitational transition dynamics.*

Proof. Each jewel in Indra's net is a node in $\mathcal{G}$: a massive concentration of matter anchored by a supermassive black hole, at the intersection of cosmic filaments. Each thread connecting jewels is an edge in $\mathcal{G}$: a cosmic filament along which matter and energy flow.

Each jewel reflects all others because the gravitational field of each node is shaped by the entire nodal network: the Koopman linearization of the gravitational transition dynamics produces a representation in which the state of each node is a linear function of the states of all other nodes — precisely the reflection structure of Indra's net.

The net has no privileged center because the cosmic graph has no privileged node: the Nabaala Theorem establishes that every embedded observer is equidistant from the center of S^3, which is the organizing principle of the geometry but is accessible to no embedded point. The center of the net — the view from nowhere — is not a jewel. It is the geometry that makes the net possible.

The infinite regress of reflections within reflections is the simplicial tower: each level of the clique complex of the cosmic graph reflecting the relational structure of the levels above and below it, without a terminus in the upward direction for a sufficiently rich graph topology. $\quad\square\qquad\square$

Remark 7.2. The Buddhist tradition described this geometry two and a half millennia before Fock mapped the hydrogen atom onto S^3, before the Nabaala Theorem located the observational ceiling, before the cosmic web was observed in galaxy surveys. The language was not waiting to be invented. It was waiting to catch back up to what the intuition already knew. The mathematics is the recovery, not the discovery.

8 The Unified Theorem

Theorem 8.1 (Cosmic Graph Theorem). *The following chain of identifications holds within the Imagination Machine framework:*

1. *The cosmic web is the graph $\mathcal{G} = (V, E, F)$ drawn on the three-dimensional spatial manifold of the universe, one dimension above the two-sphere observational boundary of the embedded observer.*
2. *The nodes V are the massive concentrations of matter at cosmic filament intersections, anchored by supermassive black holes acting as topological gates in the sense of TIM XXVIII.*
3. *The edges E are the cosmic filaments: the relational structure of the cosmic graph, gravitationally present at every scale but not encodable as local particle content on the observer's S^2 boundary. This is what is observed as dark matter.*
4. *The faces F are the cosmic voids: the bubbles enclosed by the filamentary network, through whose expanding topology the cosmological bubble passes. The horn-filling dynamics of this passage produce accelerating separation between nodes. This is what is observed as dark energy.*
5. *The Nabaala observational limit establishes that the observer cannot directly encode the edge structure as local particle content, because it is one dimension above the observer's encodable boundary. Dark matter detection experiments will continue to find no particle signal for this structural reason.*
6. *Indra's net is a precise geometric description of $\mathcal{G}$ under Koopman linearization of the gravitational dynamics: each node reflecting all others, no privileged center, infinite relational depth.*

Remark 8.2. The Cosmic Graph Theorem does not require new physics. It requires the recognition that the topology of spacetime is the fundamental dynamical variable — the resolution condition for quantum gravity established in TIM XXXI — and that the cosmic web is the graph drawn on the three-dimensional manifold in exactly the structural position that the quotient graph occupies on the two-sphere boundary of the embedded observer.

The framework requires no modification to accommodate this result. The extension is the natural consequence of asking whether the horn-filling principle, already established from cognition to cosmological spacetime, continues one dimension up.

It does.

9 Open Questions

Several questions follow immediately from the Cosmic Graph Theorem and remain open.

Quantitative predictions. The identification of dark matter with the edge structure of the cosmic graph generates quantitative predictions: the distribution of dark matter halos should trace the filamentary edge structure of the cosmic web, the ratio of dark matter to baryonic matter should reflect the ratio of edge structure to nodal content in the cosmic graph, and the absence of dark matter signals in direct detection experiments should be exact rather than merely consistent with current sensitivity limits. Each of these is testable against existing observational data.

The cosmological constant. The identification of dark energy with horn-filling dynamics reframes the cosmological constant problem as a topological question. The value of Λ should be derivable from the topology of the cosmic graph — from the ratio of void volume to filamentary volume in

the observable universe. Whether this derivation produces the observed value is an open quantitative question.

The transition redshift. The transition from decelerating to accelerating expansion at redshift $z \approx 0.7$ should correspond to the transition from node-dominated to void-dominated topology in the expanding bubble's path through the cosmic graph. Whether the observed transition redshift is consistent with the topological transition predicted by the graph structure is a testable prediction.

The graph genus. The cosmic graph is drawn on a three-dimensional manifold. The Nabaala Theorem of General Subject-Relativity applies to two-dimensional boundaries. The generalization of the Heawood bound to three-dimensional manifolds — and its implications for the self-classification depth of cosmological observers — is an open mathematical question.

The light tube. The embedded observer on Earth sits between the Sun and the galactic center, Sagittarius A*. Both are effective topological gates. The observer is literally in the light tube between two black holes — one stellar, one supermassive — whose combined gravitational topology shapes the conditions under which biological embedded epistemic systems evolved. Whether this specific topological position is necessary for the evolution of observers capable of discovering the framework is an open question that the framework itself cannot answer from within.

10   Conclusion

The Imagination Machine series has established the horn-filling principle at every scale examined: from individual cognition to neural dynamics, from simplicial completion to cosmological space-time, from the foundations of mathematics to the incompatibility of quantum field theory and general relativity.

The present paper extends the principle one dimension up. The cosmic web is the graph drawn on the three-dimensional spatial manifold of the universe. Dark matter is the relational edge structure of that graph — not a substance, but the net itself. Dark energy is the horn-filling dynamics of the cosmological bubble expanding through the void faces of that graph — not a substance, but the topology in motion.

Both phenomena are structural consequences of the graph topology of spacetime. Neither requires new physics. Both require the recognition that topology is the fundamental dynamical variable — which is precisely the resolution condition for quantum gravity established in TIM XXXI.

The cosmic web is Indra's net. The geometry was always already there. The mathematics has caught back up.

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