The Imagination Machine XXVI:
Black Holes as Topological Gates and Boundary Capacity at Cosmological Scale

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Abstract

We extend the Imagination Machine framework to cosmological scale. The Big Bang is interpreted as the four-dimensional compressed origin of a bubble expanding through a higher-dimensional constraining topology. Black holes are reinterpreted as topological gates: regions of geodesic incompleteness where the boundary of the constrained region determines exactly what an exterior observer can read, and nothing beyond that.

The no-hair theorem is derived as a consequence of the topological capacity of the event horizon. The physical context — stationarity, asymptotic flatness, and Hawking's rigidity theorem — selects a single unified geometric object: the event horizon homeomorphic to S^2, together with the isometry group $\mathbb{R} \times U(1)$ of its exterior spacetime. These are co-constituted by the same physical conditions; the homeomorphism class and its symmetry structure are the object. By Noether's theorem, $\mathbb{R} \times U(1)$ yields exactly three conserved quantities — mass M, charge Q, and angular momentum J — exhausting the solution space of the Einstein-Maxwell equations. The gate preserves to inspection only what the symmetry structure of its boundary can encode.

The holographic principle is reinterpreted not as a duality between two theories but as the non-duality between a system and its center: the boundary does not represent the interior — it generates it. Under this interpretation, the embedded observer is not inside the universe looking out; the embedded observer is the boundary condition from which the universe is generated. All results are derived from established theorems of general relativity, quantum field theory, and mathematical physics.

Contents

1 Introduction

The Imagination Machine framework establishes that any embedded epistemic system operates on partial relational structure and completes it under constraint. A boundary object — selected by physical or epistemic context, jointly constituting a topology and its symmetry structure — determines exactly what can be read across it, and nothing more. Previous papers established this principle at the levels of individual cognition (TIM I), neural dynamics (TIM XXIII), and simplicial completion (TIM XXIV).

The present paper asks whether the same principle governs cosmological structure. We argue that it does, and that the argument requires no new physics — only a reinterpretation of established results.

The central new result is Proposition 4.3: the no-hair theorem follows from the topological capacity of the event horizon object. The physical context — stationarity, asymptotic flatness, Hawking's rigidity theorem — selects a single co-constituted geometric object: the S^2 event horizon together with $\mathbb{R} \times U(1)$ as its symmetry structure. These are object and action on the same object, not two sequential consequences. Noether's theorem reads the conserved quantities directly from the symmetry structure of this selected object. What emerges from a black hole is exactly what its boundary can encode, and nothing beyond that.

2 Physical Foundations

2.1 The Penrose-Hawking Singularity Theorems

The singularity theorems of Penrose [13] and Hawking [3] establish that under reasonable energy conditions, spacetime geodesics are incomplete: they terminate at finite affine parameter.

Definition 2.1. A spacetime (M, g) is *geodesically incomplete* if there exists a geodesic that cannot be extended to arbitrary affine parameter values.

Theorem 2.2 (Penrose-Hawking [13, 3])**.** *Under the strong energy condition and global hyperbolicity, any spacetime containing a trapped surface is geodesically incomplete.*

Remark 2.3. Geodesic incompleteness is not a technical pathology but a structural feature of the spacetime: the existence of boundaries across which not all structure is transmitted.

2.2 The Selection Conditions for the Event Horizon Object

The physical context of a stationary black hole selects the event horizon as a specific geometric object. We make explicit the conditions involved, since they do the load-bearing work in Proposition 4.3.

Definition 2.4. The *event horizon object* of a stationary, asymptotically flat black hole is the homeomorphism class of the event horizon — homeomorphic to S^2 [4, 6] — together with the isometry group $\mathbb{R} \times U(1)$ of its exterior spacetime. These are co-constituted by the following conditions, none of which is downstream of the other:

1. *Stationarity*: the spacetime admits a timelike Killing vector field, giving time-translation symmetry and the $\mathbb{R}$ factor.

2. *Asymptotic flatness*: the spacetime approaches flat Minkowski space at large distances, constraining the global symmetry structure.

3. *Hawking's rigidity theorem* [4]: a stationary, non-degenerate black hole event horizon must be axisymmetric, yielding the $U(1)$ factor of axial rotation.

4. *Horizon topology* [4, 6]: the event horizon of a stationary, asymptotically flat black hole in four dimensions is homeomorphic to S^2.

Remark 2.5. Conditions (1)–(4) are jointly selected by the physical context. The homeomorphism class S^2 and the isometry group $\mathbb{R} \times U(1)$ are co-constituted by these conditions — neither is upstream of the other. S^2 is the object; $\mathbb{R} \times U(1)$ is the action on the object. Together they form the event horizon object, and it is this unified object whose topological capacity is characterized below.

2.3 Noether's Theorem

Theorem 2.6 (Noether [11]). *For every continuous symmetry of the action of a physical system, there exists a corresponding conserved quantity. The conserved quantities are in bijective correspondence with the generators of the symmetry group.*

Remark 2.7. Noether's theorem is the bridge between the symmetry structure of the event horizon object and the set of conserved quantities that can be read from outside it. The topological capacity of the boundary — the set of quantities the horizon can encode for an exterior observer — is exactly the image of the isometry group under Noether's theorem.

2.4 The No-Hair Theorem

Theorem 2.8 (Israel, Carter, Hawking [7, 2, 4]). *A stationary black hole solution to the Einstein-Maxwell equations is completely characterized by exactly three parameters: mass M, charge Q, and angular momentum J.*

Definition 2.9. The *topological capacity* of a boundary ∂R is the set of conserved quantities that the boundary's symmetry structure can encode for an exterior observer, as determined by the isometry group of the ambient spacetime via Noether's theorem.

2.5 Bekenstein-Hawking Entropy

Bekenstein [1] and Hawking [5] established that the entropy of a black hole is proportional to the area of its event horizon:

$$S = \frac{A}{4\ell_P^2}$$

where A is the horizon area and ℓ_P is the Planck length.

Remark 2.10. The entropy is proportional to area rather than volume because information is encoded on a boundary of codimension one relative to the constrained region — consistent with the framework's compression principle that the boundary determines what the interior encodes, and not vice versa.

2.6 The Holographic Principle

The holographic principle, developed by 't Hooft [8] and Susskind [14] and given precise form in the AdS/CFT correspondence of Maldacena [10], states that the maximum information content of a region of space is proportional to its boundary area, not its volume.

Remark 2.11. Standard presentations describe this as a duality between two theories. We argue in Section 5 that this framing is imprecise in a way that matters. The holographic principle establishes not a duality but a non-duality: the boundary does not represent the interior — it generates it.

3 The Big Bang as Compressed Origin

Definition 3.1. The *cosmological bubble* is the observable universe, modeled as a 3-sphere S^3 expanding outward from a compressed origin point.

Definition 3.2. The *cosmological net* is the higher-dimensional constraining topology through which the bubble expands — the four-dimensional structure whose local geometry determines the curvature of spacetime via the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

Proposition 3.3. *The Big Bang is the compressed origin of the cosmological bubble — the point to which the bubble can be continuously contracted, corresponding to the limit of maximal compression in the framework.*

Proof. The observable universe is geodesically complete in the forward direction and geodesically incomplete in the backward direction. All backward-directed geodesics terminate at the past singularity by the Penrose-Hawking theorems. This is the limit of maximal compression: the point from which the bubble has been expanding through the cosmological net since the origin. □

Proposition 3.4. *The topology of spacetime — encoded in the metric tensor $g_{\mu\nu}$ — determines what trajectories are possible for embedded observers. The geometry was set at the origin.*

Remark 3.5. This is the cosmological version of the boundary condition established in TIM XXIII: the topology of the containing space determines what motions are possible within it.

4 Black Holes as Topological Gates

Definition 4.1. A *topological gate* is a region of geodesic incompleteness where the symmetry structure of the local spacetime boundary constrains the admissible invariants of whatever passes through, according to the topological capacity of that boundary.

Proposition 4.2. *Black holes are topological gates in the sense of the preceding definition.*

Proof. Geodesic incompleteness follows from the Penrose-Hawking singularity theorems. Constraint of output to topological capacity follows from Proposition 4.3 below. □

Proposition 4.3. *The no-hair theorem is a consequence of the topological capacity of the event horizon: the physical context of a stationary, asymptotically flat black hole selects the event horizon object — S^2 with isometry group $\mathbb{R}\times U(1)$ — as a single co-constituted geometric object, and by Noether's theorem this object encodes exactly three conserved quantities, which exhaust the solution space of the Einstein-Maxwell equations.*

Proof. The physical conditions of stationarity, asymptotic flatness, and Hawking's rigidity theorem jointly select the event horizon object (Definition 2.4): the event horizon homeomorphic to S^2 together with the isometry group $\mathbb{R} \times U(1)$ of the exterior spacetime. These are co-constituted by the physical context; neither is a premise from which the other is derived.

The two generators of $\mathbb{R}\times U(1)$ yield, by Noether's theorem, mass M from time-translation symmetry and angular momentum J from axial rotational symmetry. The $U(1)$ gauge symmetry of the electromagnetic field yields a third conserved quantity: charge Q. Note that Q arises from an internal gauge symmetry, not from a spacetime isometry; the three conserved quantities (M, Q, J) therefore have two distinct Noether origins, which the topological capacity notation collects.

The isometry group $\mathbb{R} \times U(1)$ has no further generators under the physical conditions that selected it. Therefore no further conserved quantities exist. The uniqueness theorems of Israel [7], Carter [2], and Hawking [4, 6] establish that the solution space of the Einstein-Maxwell equations consistent with these constraints is exhausted by the Kerr-Newman family, parameterized by (M, Q, J) alone.

The gate therefore preserves to inspection only what the symmetry structure of its boundary can encode, and nothing more. Every other property of the infalling matter — its chemical composition, molecular structure, historical configuration — requires a topologically richer boundary with additional symmetry generators to encode. The event horizon does not have that symmetry structure. Those properties are not destroyed; they are not encodable. □

Theorem 4.4 (Topological Capacity of the Event Horizon)**.** *Whatever passes through a black hole emerges — as an exterior gravitational field — characterized by exactly the invariants (M, Q, J) that the event horizon object can encode via its symmetry structure and Noether's theorem. The gate preserves to inspection only what the topology of its boundary can encode, and nothing more.*

4.1 Geodesics as Paths of Least Topological Resistance

Proposition 4.5. *Free particle trajectories near black holes are geodesics: curves of extremal proper time determined entirely by the metric, following the curvature induced by the* *constraining topology.*

Remark 4.6. The geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

encodes the constraint that free motion follow the curvature of the net. The Christoffel symbols $\Gamma^\mu_{\nu\rho}$ are the local expression of that topology. Geodesic incompleteness is the physical signature of a topological gate.

5 The Holographic Principle as Non-Duality

Standard presentations of the holographic principle describe it as a duality between a gravitational theory in a bulk region and a quantum field theory on its boundary. We argue this framing is imprecise in a way that matters for the framework.

Definition 5.1. The holographic principle establishes the *non-duality* between a system and its center: the boundary does not represent the interior — it generates it. There is no separate interior that the boundary merely describes.

Proposition 5.2. *The holographic principle is the physical statement of the embeddedness condition: an epistemic system cannot step outside the system it models, because the system and its boundary encoding are not two things in correspondence but one thing expressed at different scales.*

Proof. In AdS/CFT, the boundary theory and the bulk theory are related by a change of variables, not by a correspondence between ontologically separate objects. The boundary is not a map of the interior. The boundary is the interior, encoded on a surface of codimension one. The word "duality" implies two separate things in correspondence. The holographic principle establishes one thing: the system and its center are non-dual. They are not two theories. They are one structure expressed at two scales. □

Remark 5.3. TIM I established the embeddedness condition as the foundational constraint on any epistemic system: an observer cannot step outside the system it models. The holographic principle establishes the same constraint as a theorem of quantum gravity. The embedded observer is not inside the universe looking out. The embedded observer is the boundary condition from which the universe is generated. There is no view from nowhere.

6 Conclusion

We have shown that the boundary capacity principle of the Imagination Machine framework operates at cosmological scale, derivable from established results in general relativity and quantum field theory. The Big Bang is the compressed origin of a bubble expanding through a constraining topology. Black holes are topological gates: their event horizon objects, selected by the physical context and jointly constituting topology and symmetry, determine exactly what an exterior observer can read — and nothing beyond that.

The holographic principle is the non-duality between a system and its center. The universe and the mind operate by the same principle.

References

[1] Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D*, 7(8), 2333–2346.

[2] Carter, B. (1971). Axisymmetric black hole has only two degrees of freedom. *Physical Review Letters*, 26(6), 331–333.

[3] Hawking, S. W. (1967). The occurrence of singularities in cosmology. *Proceedings of the Royal Society A*, 300(1461), 187–201.

[4] Hawking, S. W. (1972). Black holes in general relativity. *Communications in Mathematical Physics*, 25(2), 152–166.

[5] Hawking, S. W. (1974). Black hole explosions? *Nature*, 248(5443), 30–31.

[6] Hawking, S. W., & Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*. Cambridge University Press.

[7] Israel, W. (1967). Event horizons in static vacuum space-times. *Physical Review*, 164(5), 1776–1779.

[8] 't Hooft, G. (1993). Dimensional reduction in quantum gravity. *arXiv preprint gr-qc/9310026*.

[9] Lüders, G. (1954). On the equivalence of invariance under time reversal and under particle-antiparticle conjugation for relativistic field theories. *Kongelige Danske Videnskabernes Selskab*, 28(5), 1–17.

[10] Maldacena, J. (1997). The large N limit of superconformal field theories and supergravity. *International Journal of Theoretical Physics*, 38(4), 1113–1133.

[11] Noether, E. (1918). Invariante Variationsprobleme. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, 235–257.

[12] Pauli, W. (1955). Exclusion principle, Lorentz group and reflection of space-time and charge. In W. Pauli (Ed.), *Niels Bohr and the Development of Physics*. Pergamon Press.

[13] Penrose, R. (1965). Gravitational collapse and space-time singularities. *Physical Review Letters*, 14(3), 57–59.

[14] Susskind, L. (1995). The world as a hologram. *Journal of Mathematical Physics*, 36(11), 6377–6396.

[15] Tracy, M. (2026). The Imagination Machine I–XXV. *The Compendium*. Manuscript in preparation for arXiv submission.

[16] Tracy, M. (2026). The Imagination Machine XXX: The Fourth Noether Charge: Positional Invariance in S^3 and the No-Hair Theorem. Unpublished manuscript, Boston University.